

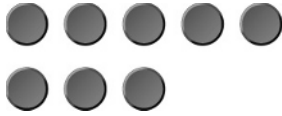
LESSON
1-1

Adding Integers with the Same Sign

Practice and Problem Solving: A/B

Find each sum. White counters are positive. Black counters are negative.

1. $-5 + (-3)$

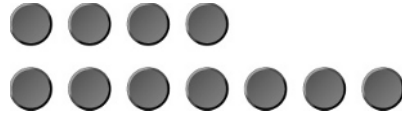


a. How many counters are there? _____

b. Do the counters represent positive or negative integers? _____

c. $-5 + (-3) =$ _____

2. $-4 + (-7)$



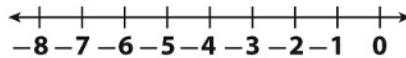
a. How many counters are there? _____

b. Do the counters represent positive or negative integers? _____

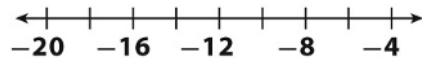
c. $-4 + (-7) =$ _____

Model each addition problem on the number line to find each sum.

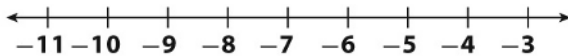
3. $-4 + (-2) =$ _____



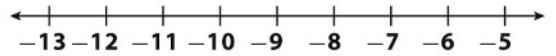
4. $-5 + (-5) =$ _____



5. $-3 + (-6) =$ _____



6. $-7 + (-5) =$ _____



Find each sum.

7. $-7 + (-1) =$ _____

8. $-5 + (-4) =$ _____

9. $-36 + (-17) =$ _____

10. $-51 + (-42) =$ _____

11. $98 + 126 =$ _____

12. $-20 + (-75) =$ _____

13. $-350 + (-250) =$ _____

14. $-110 + (-1200) =$ _____

Solve.

15. A construction crew is digging a hole. On the first day, they dug a hole 3 feet deep. On the second day, they dug 2 more feet. On the third day, they dug 4 more feet. Write a sum of negative numbers to represent this situation. Find the total sum and explain how it is related to the problem.

LESSON

1-1

Adding Integers with the Same Sign***Practice and Problem Solving: C*****Solve.**

1. A grocery sells green apples and red apples. On Monday, the store put 500 of each kind of apple on display. That day, the store sold 42 red apples and 57 green apples. On Tuesday, the store sold 87 red apples and 75 green apples. On Wednesday, the store sold 29 red apples and 38 green apples.

- a. Write an addition expression using negative integers to show the number of red apples the store sold.

- b. Write an addition expression using negative integers to show the number of green apples the store sold.

- c. Did the store have more red apples or green apples left over? Explain.

2. A hotel has 18 floors. The hotel owner believes the number 13 is unlucky. The first 12 floors are numbered from 1 to 12. Floor 13 is numbered 14, and the remaining floors are numbered from 15 to 19. The hotel manager starts on the top floor of the apartment building. He rides the elevator two floors down. The doors open and a hotel guest gets in. They ride the elevator three floors down. The hotel guest gets off the elevator. The hotel manager rides the elevator the remaining floors down to the first floor.

- a. Write an addition expression using negative integers to show the number of floors the hotel manager rode down in the elevator.

- b. On what floor did the hotel guest get off the elevator? Explain.

LESSON
1-1

Adding Integers with the Same Sign

Practice and Problem Solving: D

Find each sum. White counters are positive. Black counters are negative. The first one is done for you.

1. $5 + 2 =$



a. How many counters are there? 7

b. Do the counters represent positive or negative numbers? positive

c. $5 + 2 =$ $+7$

2. $-4 + (-6) =$



a. How many counters are there? _____

b. Do the counters represent positive or negative numbers? _____

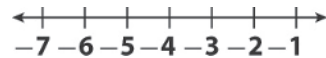
c. $-4 + (-6) =$ _____

Model each addition problem on the number line to find each sum. The first one is done for you.

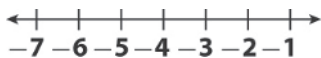
3. $-3 + (-2) =$ -5



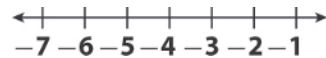
4. $-5 + (-1) =$ _____



5. $-4 + (-3) =$ _____



6. $-1 + (-6) =$ _____



Find each sum. The first one is done for you.

7. $-3 + (-1) =$ -4

8. $-6 + (-2) =$ _____

9. $-12 + (-7) =$ _____

10. $-20 + (-15) =$ _____

Solve.

11. The table shows how much money Hannah withdrew in 3 days.

Day	Day 1	Day 2	Day 3
Dollars	-5	-1	-2

Find the total amount Hannah withdrew. _____

LESSON

1-1

Adding Integers with the Same Sign**Reteach**

How do you add integers with the same sign?

Add $4 + 5$.

Step 1 Check the signs. Are the integers both positive or negative?

4 and 5 are both positive.

Step 2 Add the integers.

$$4 + 5 = 9$$

Step 3 Write the sum as a positive number.

$$4 + 5 = 9$$

Add $-3 + (-4)$.

Step 1 Check the signs. Are the integers both positive or negative?

-3 and -4 are both negative.

Step 2 Ignore the negative signs for now. Add the integers.

$$3 + 4 = 7$$

Step 3 Write the sum as a negative number.

$$-3 + (-4) = -7$$

Find each sum.

1. $3 + 6$

a. Are the integers both positive or negative? _____

b. Add the integers. _____

c. Write the sum. $3 + 6 =$ _____

2. $-7 + (-1)$

a. Are the integers both positive or negative? _____

b. Add the integers. _____

c. Write the sum. $-7 + (-1) =$ _____

3. $-5 + (-2)$

a. Are the integers both positive or negative? _____

b. Add the integers. _____

c. Write the sum. $-5 + (-2) =$ _____

4. $6 + 4$

a. Are the integers both positive or negative? _____

b. Add the integers. _____

c. Write the sum. $6 + 4 =$ _____

Find each sum.

5. $-10 + (-3) =$ _____

6. $-4 + (-12) =$ _____

7. $22 + 15 =$ _____

8. $-10 + (-31) =$ _____

9. $-18 + (-6) =$ _____

10. $35 + 17 =$ _____

LESSON
1-1

Adding Integers with the Same Sign

Reading Strategies: Use a Model

Sarah withdraws the following amounts from her bank account in 4 days.

Day	1	2	3	4
Withdrawal	-3	-5	-4	-1

Write a sum of negative integers to represent this situation.
Find the sum and explain how it is related to the problem.

You can use counters to model this problem.

To represent this situation, write: $-3 + (-5) + (-4) + (-1)$

The total number of counters is 13.

Since the counters are negative, the sum is -13 .

Over the four days, Sarah withdrew a total of \$13 from her bank account.

Answer each question.

1. What does each counter represent?

2. How do the counters help you represent the information in the table?

3. How do the counters help you find the sum?

4. Write an equation to show the total amount Sarah withdrew from her bank account.

LESSON
1-1

Adding Integers with the Same Sign

Success for English Learners

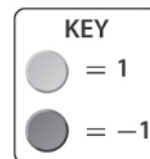
Problem 1

$$7 + 5$$

Use counters.



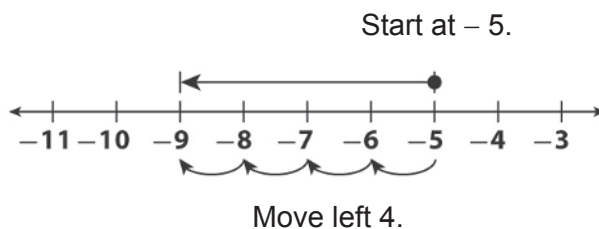
$$7 + 5 = 12$$



Problem 2

$$-5 + (-4)$$

Use a number line.



1. What kind of counters are used in Problem 1, positive or negative?

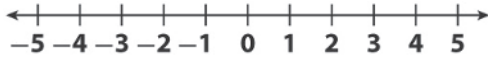
2. Why do you move left in Problem 2?

3. Write a word problem about adding integers with the same sign.

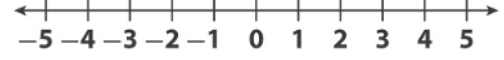
LESSON
1-2**Adding Integers with Different Signs****Practice and Problem Solving: A/B**

Show the addition on the number line. Find the sum.

1. $2 + (-3)$ _____



2. $-3 + 4$ _____



Find each sum.

3. $-4 + 9$

4. $7 + (-8)$

5. $-2 + 1$

6. $6 + (-9)$

7. $5 + (-7)$

8. $9 + (-5)$

9. $(-1) + 9$

10. $9 + (-7)$

11. $50 + (-7)$

12. $27 + (-6)$

13. $1 + (-30)$

14. $15 + (-25)$

Solve.

15. The temperature outside dropped 13°F in 7 hours. The final temperature was -2°F . What was the starting temperature?

16. A football team gains 8 yards in one play, then loses 5 yards in the next. What is the team's total yardage for the two plays?

17. Matt is playing a game. He gains 7 points, loses 10 points, gains 2 points, and then loses 8 points. What is his final score?

18. A stock gained 2 points on Monday, lost 5 points on Tuesday, lost 1 point on Wednesday, gained 4 points on Thursday, and lost 6 points on Friday.

a. Was the net change for the week positive or negative? _____

b. How much was the gain or loss? _____

LESSON

1-2

Adding Integers with Different Signs**Practice and Problem Solving: C**

Tell whether each sum will be positive or negative. Then find each sum.

1. $-3 + (-7)$

2. $14 + (-9)$

3. $-12 + 5$

4. $-3 + 8$

5. $11 + (-5)$

6. $7 + 8$

7. $-8 + 7$

8. $-2 + 3$

9. If two integers have the same sign, what is the sign of their sum?

10. When adding two integers with different signs, how do you find the sign?

Evaluate $a + b$ for the given values.

11. $a = 9, b = -24$

12. $a = -17, b = -7$

13. $a = 32, b = -19$

14. $a = -15, b = -15$

15. $a = -20, b = 20$

16. $a = -30, b = 12$

Solve.

17. The high temperature for the day dropped 7°F between Monday and Tuesday, rose 9°F on Wednesday, dropped 2°F on Thursday, and dropped 5°F on Friday. What was the total change in the daily high temperature from Monday to Friday?

18. Karen deposited \$25 in the bank on Monday, \$50 on Wednesday and \$15 on Friday. On Saturday, she took out \$40. Karen's original balance was \$100. What is her balance now?

19. Lance and Rita were tied in a game. Then Lance got these scores: 19, -7 , 3, -11 , 5. Rita got these scores: 25, -9 , 5, -9 , 8. Who had the higher score? How much higher was that higher score?

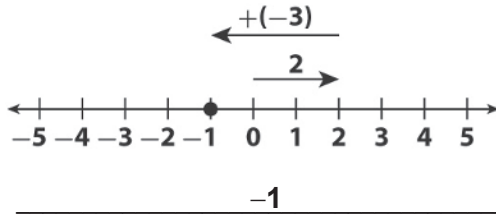
LESSON
1-2

Adding Integers with Different Signs

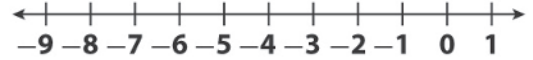
Practice and Problem Solving: D

Show the addition on the number line. Then write the sum. The first one is done for you.

1. $2 + (-3)$



2. $-3 + (-4)$



Find each sum. The first one is done for you.

3. $4 + (-9)$

_____ **-5** _____

4. $7 + (-8)$

5. $-2 + 1$

6. $5 + 7$

7. $9 + (-5)$

8. $-1 + 9$

9. $2 + (-7)$

10. $-6 + (-4)$

11. $-15 + 9$

Solve. The first one is done for you.

12. The temperature dropped 12°F in 8 hours. If the final temperature was -7°F , what was the starting temperature?

5°F

13. At 3 P.M., the temperature was 9°F . By 11 P.M., it had dropped 31°F . What was the temperature at 11 P.M.?

14. A submarine submerged at a depth of -40 feet dives 57 feet more. What is the new depth of the submarine?

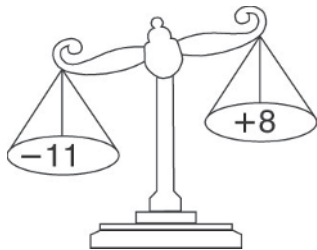
15. An airplane cruising at 20,000 feet drops 2,500 feet in altitude. What is the airplane's new altitude?

LESSON
1-2

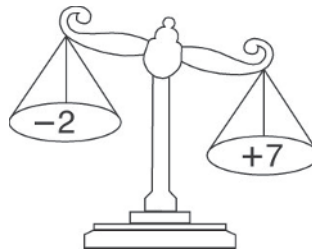
Adding Integers with Different Signs

Reteach

This balance scale “weighs” positive and negative numbers. Negative numbers go on the left of the balance, and positive numbers go on the right.



Find $-11 + 8$.
The scale will tip to the left side because the sum of -11 and $+8$ is negative.
 $-11 + 8 = -3$



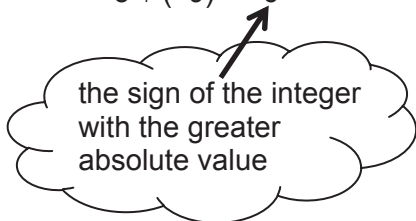
Find $-2 + 7$.
The scale will tip to the right side because the sum of -2 and $+7$ is positive.
 $-2 + 7 = 5$

Find $3 + (-9)$.

1. Should you add or subtract 3 and 9? Why?

2. Is the sum positive or negative? _____

$$3 + (-9) = -6$$



Find the sum.

3. $7 + (-3) =$ _____

4. $-2 + (-3) =$ _____

5. $-5 + 4 =$ _____

6. $-3 + (-1) =$ _____

7. $-7 + 9 =$ _____

8. $4 + (-9) =$ _____

9. $16 + (-7) =$ _____

10. $-21 + 11 =$ _____

11. $-12 + (-4) =$ _____

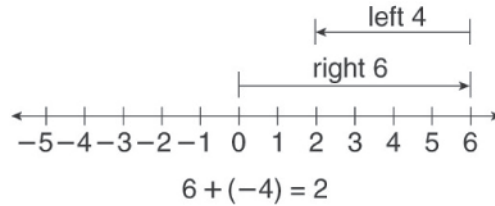
12. When adding 3 and -9 , how do you know that the sum is negative?

LESSON
1-2

Adding Integers with Different Signs

Reading Strategies: Use Graphic Aids

Randy’s football team had the ball on its own zero yard line. On their first play they gained 6 yards. On the second play they lost 4 yards. On what yard line is the ball now?



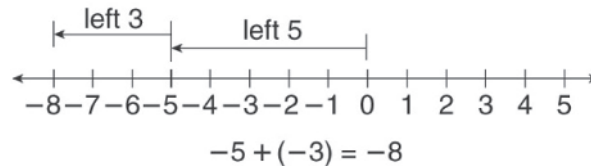
Use the number line to help you answer the questions.

1. On which number do you begin? _____
2. In which direction do you move first? How many places do you move?

3. In which direction do you move next? How many places do you move?

4. At which number do you end up? _____

The temperature was zero degrees. Two hours later, the temperature went down 5 degrees. Then, the temperature went down another 3 degrees. What was the final temperature?



Use the number line to help you answer the questions.

5. On which number do you begin? _____
6. In which direction do you move first? How many spaces?

7. In which direction do you move next? How many spaces?

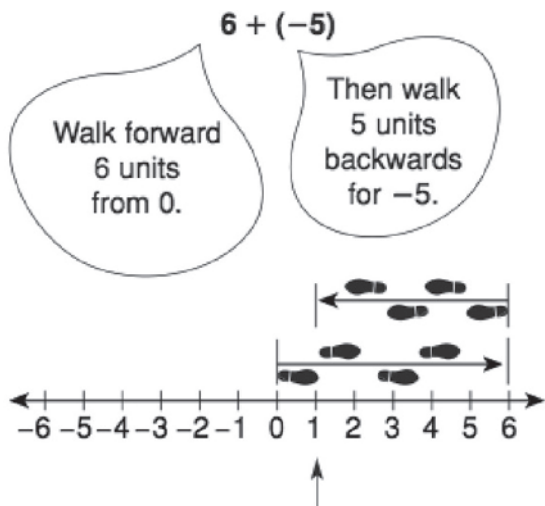
8. At which number do you end up? _____

LESSON
1-2

Adding Integers with Different Signs

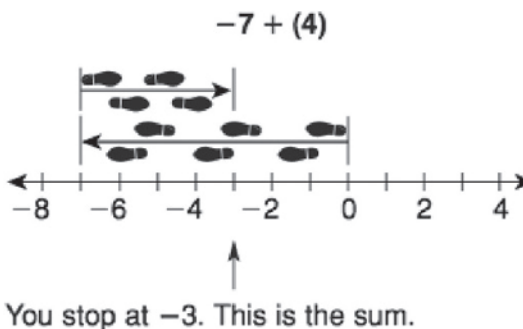
Success for English Learners

Problem 1



You stop at 1. This is the sum.

Problem 2



1. The sum of $x + y$ is to the left of x on a number line. Is y a positive number or a negative number?

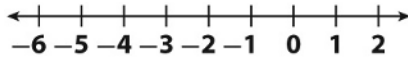
2. Based on Problems 1 and 2, does the addition of integers always mean the sum is positive? Explain.

3. Is the sum of $3 + (-9)$ positive or negative?

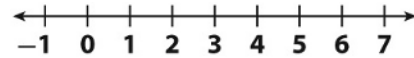
4. Is the sum of $13 + (-11)$ positive or negative?

LESSON
1-3**Subtracting Integers****Practice and Problem Solving: A/B****Show the subtraction on the number line. Find the difference.**

1. $-2 - 3$



2. $5 - (-1)$



Find the difference.

3. $-6 - 4$

4. $-7 - (-12)$

5. $12 - 16$

6. $5 - (-19)$

7. $-18 - (-18)$

8. $23 - (-23)$

9. $-10 - (-9)$

10. $29 - (-13)$

11. $9 - 15$

12. $-12 - 14$

13. $22 - (-8)$

14. $-16 - (-11)$

Solve.

15. Monday's high temperature was 6°C . The low temperature was -3°C .
What was the difference between the high and low temperatures?

16. The temperature in Minneapolis changed from -7°F at 6 A.M. to 7°F at noon. How much did the temperature increase?

17. Friday's high temperature was -1°C . The low temperature was -5°C .
What was the difference between the high and low temperatures?

18. The temperature changed from 5°C at 6 P.M. to -2°C at midnight. How much did the temperature decrease?

19. The daytime high temperature on the moon can reach 130°C . The nighttime low temperature can get as low as -110°C . What is the difference between the high and low temperature?

LESSON
1-3**Subtracting Integers****Practice and Problem Solving: C**

For each set of values find $x - y$. Answer the questions that follow.

1. $x = 14, y = -2$

2. $x = -11, y = 11$

3. $x = -8, y = -15$

4. $x = -9, y = -9$

5. $x = 9, y = -20$

6. $x = 0, y = -9$

7. $x = 9, y = 11$

8. $x = -1, y = -1$

9. $x = -5, y = 5$

10. If x and y are both positive, when is $x - y$ negative? _____11. If x and y are both negative, when is $x - y$ positive? _____**Solve.**

12. The temperature changed from 7°F at 6 P.M. to -5°F at midnight. What was the difference between the high and low temperatures? What was the average change in temperature per hour?

13. The lowest point in the Pacific Ocean is about $-11,000$ meters. The lowest point in the Atlantic Ocean is about $-8,600$ meters. Which ocean has the lower point? How much lower?

14. At 11,560 feet above sea level, Climax, Colorado, is the highest town in the United States. The lowest town is Calipatria, California, at 185 feet below sea level. Express both of these distances as integers and tell which is closer to sea level. How much closer to sea level is the town that is closer?

Use the table for 15–16.

Temperatures at a Ski Resort

Day	High	Low
Saturday	8°F	-3°F
Sunday	6°F	-2°F

15. On which day was the difference in temperature greater? _____

16. How much greater was the difference one day than the other? _____

LESSON
1-3

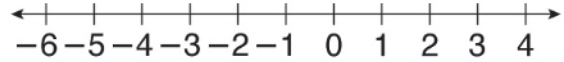
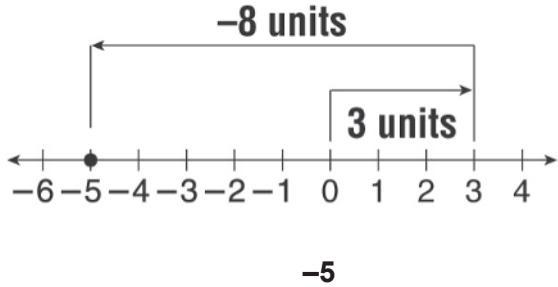
Subtracting Integers

Practice and Problem Solving: D

Show the subtraction on the number line. Then write the difference.
The first one is done for you.

1. $3 - 8$

2. $-5 - (-1)$



Find each difference. The first one is done for you.

3. $-3 - 4$

4. $-7 - (-2)$

5. $12 - 6$

-7

6. $-8 - 8$

7. $-5 - (-5)$

8. $-1 - (-2)$

9. $8 - 1$

10. $7 - (-9)$

11. $-3 - 8$

Solve. The first one is done for you.

12. The daytime temperature on the planet Mercury can reach 430°C . The nighttime temperature can drop to -180°C . What is the difference between these temperatures?

610°C

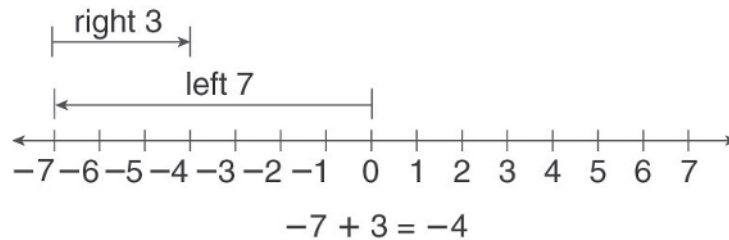
13. An ice cream company made a profit of \$24,000 in 2011. The same company had a loss of \$11,000 in 2012. What is the difference between the company's financial results for 2011 and 2012?

14. The high temperature on Saturday day was 6°F . The low temperature was -3°F . What was the difference between the high and low temperatures for the day?

LESSON
1-3**Subtracting Integers****Reading Strategies: Use Graphic Aids**

Brett borrowed \$7 from his father to buy a cap. He paid back \$3.
How much money does Brett have now?

A number line can help you picture this situation.

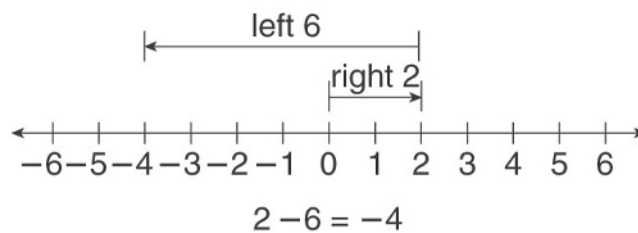


1. Beginning at 0, in which direction will you move first? _____
2. How many places? _____
3. In which direction do you move next? _____
4. How many places? _____
5. On what number do you end? _____

Brett does not have any more money. He owes his dad \$4. He has negative \$4.

Sally and her friends made up a game with points. You can either win or lose up to ten points on each round of the game. After the first round, Sally's team had 2 points. In the second round, they lost 6 points. By how many points was Sally's team down after the second round?

The number line will help you picture the problem.



6. Beginning at 0, in which direction will you move first? How many places?

7. Which direction will you move next? How many places?

8. On what number do you end? _____

LESSON
1-3

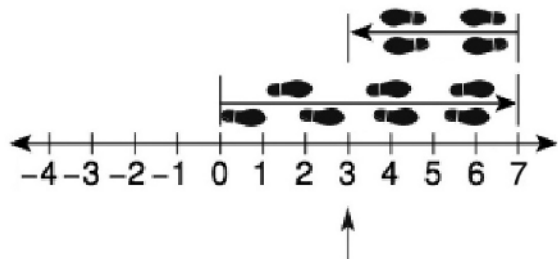
Subtracting Integers

Success for English Learners

Problem 1

What is the difference?

$$7 - 4$$

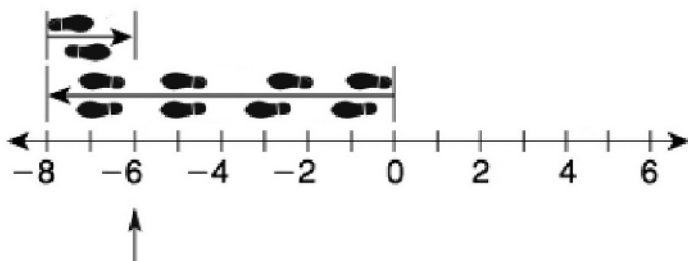


You stop at 3. This is the difference.

Problem 2

What is the difference?

$$-8 - (-2)$$



You stop at -6. This is the difference.

1. If $x > 0$ and $x > y$, is the difference $x - y$ positive or negative?

2. If $x > 0$ and $y > x$, is the difference $x - y$ positive or negative?

LESSON
1-4**Applying Addition and Subtraction of Integers****Practice and Problem Solving: A/B**

Write an expression to represent the situation. Then solve by finding the value of the expression.

1. Owen is fishing from a dock. He starts with the bait 2 feet below the surface of the water. He reels out the bait 19 feet, then reels it back in 7 feet. What is the final position of the bait relative to the surface of the water?

2. Rita earned 45 points on a test. She lost 8 points, earned 53 points, then lost 6 more points. What is Rita's final score on the test?

Find the value of each expression.

3. $-7 + 12 + 15$

4. $-5 - 9 - 13$

5. $40 - 33 + 11$

6. $57 + 63 - 10$

7. $-21 - 17 + 25 + 65$

8. $12 + 19 + 5 - 2$

Compare the expressions. Write $<$, $>$ or $=$.

9. $-15 + 3 - 7$ $-9 - 1 + 16$

10. $31 - 4 + 6$ $-17 + 22 - 5$

Solve.

11. Anna and Maya are competing in a dance tournament where dance moves are worth a certain number of points. If a dance move is done correctly, the dancer earns points. If a dance move is done incorrectly, the dancer loses points. Anna currently has 225 points.

- a. Before her dance routine ends, Anna earns 75 points and loses 30 points. Write and solve an expression to find Anna's final score.

- b. Maya's final score is 298. Which dancer has the greater final score?

LESSON

1-4

Applying Addition and Subtraction of Integers***Practice and Problem Solving: C***

Write an expression to represent the situation. Then solve by finding the value of the expression.

1. Jana is doing an experiment. She is on a dock that is 10 feet above the surface of the water. Jana drops the weighted end of a fishing line 35 feet below the surface of the water. She reels out the line 29 feet, and then reels it back in 7 feet. What is the final distance between Jana and the end of the fishing line?
-

2. Kirsten and Gigi are riding in hot air balloons. They start 500 feet above the ground. Kirsten's balloon rises 225 feet, falls 105 feet, and then rises 445 feet. Every time Kirsten's balloon travels up or down, Gigi's balloon travels 15 feet farther in the same direction. Then both balloons stop moving so a photographer on the ground can take a picture.

- a. Find Kirsten's final position relative to the ground.
-

- b. Is Kirsten or Gigi closer to the ground when the photographer takes the picture?
-

3. In a ring-toss game, players get points for the number of rings they can toss and land on a colored stake. They earn 20 points for landing on a red stake and 30 points for landing on a blue stake. They lose 10 points each time they miss. The table shows the number of rings tossed by David and Jon during the game.

- a. Write and evaluate an expression that represents David's total score.

Player	Red	Blue	Miss
David	2	3	3
Jon	3	2	2

- b. Who scored more points during the game?
-

LESSON
1-4

Applying Addition and Subtraction of Integers

Practice and Problem Solving: D

Write an expression to represent the situation. Then solve by finding the value of the expression. The first one is done for you.

1. Jeremy is fishing from a dock. He starts with the bait 2 feet below the surface of the water. He lowers the bait 9 feet, then raises it 3 feet. What is the final position of the bait relative to the surface of the water?

$-2 - 9 + 3 = -8$; 8 feet below the surface of the water

2. Rita earned 20 points on a quiz. She lost 5 points for poor penmanship, then earned 10 points of extra credit. What is Rita's final score on the quiz?
-

Find the value of each expression. The first one is done for you.

3. $-7 + 1 + 5$

-1

4. $-5 - 9 - 10$

5. $40 - 30 + 10$

6. $2 + 8 - 19$

7. $-12 + 14 + 6$

8. $50 + 60 - 10$

Compare the expressions. Write $<$, $>$, or $=$.

9. $-20 + 5 - 10$ ○ $-10 - 11 + 30$

10. $-10 + 40 - 5$ ○ $25 - 15 + 3$

Solve.

11. Angela is competing in a dance competition. If a dance move is done correctly, the dancer earns points. If a dance move is done incorrectly, the dancer loses points. Angela currently has 200 points. Angela then loses 30 points and earns 70 points. Write and evaluate an expression to find Angela's final score.
-

LESSON
1-4

Applying Addition and Subtraction of Integers

Reteach

How do you find the value of expressions involving addition and subtraction of integers?

Find the value of $17 - 40 + 5$.

$(17 + 5) - 40$ Regroup the integers with the same sign.

$22 - 40$ Add inside the parentheses.

$22 - 40 = -18$ Subtract.

So, $17 - 40 + 5 = -18$.

Find the value of each expression.

1. $10 - 19 + 5$

a. Regroup the integers.

b. Add and subtract.

c. Write the sum. $10 - 19 + 5 =$ _____

2. $-15 + 14 - 3$

a. Regroup the integers.

b. Add and subtract.

c. Write the sum. $-15 + 14 - 3 =$ _____

3. $-80 + 10 - 6$

a. Regroup the integers.

b. Add and subtract.

c. Write the sum. $-80 + 10 - 6 =$ _____

4. $7 - 21 + 13$

a. Regroup the integers.

b. Add and subtract.

c. Write the sum. $7 - 21 + 13 =$ _____

5. $-5 + 13 - 6 + 2$

a. Regroup the integers.

b. Add and subtract.

c. Write the sum. $-5 + 13 - 6 + 2 =$ _____

6. $18 - 4 + 6 - 30$

a. Regroup the integers.

b. Add and subtract.

c. Write the sum. $18 - 4 + 6 - 30 =$ _____

LESSON
1-4**Applying Addition and Subtraction of Integers****Reading Strategies: Analyze Information**

Read the problem below.

Angelo is riding in a hot air balloon. The balloon begins at 700 feet above the ground. It drops 200 feet, rises 500 feet, and then drops 100 feet. Write and evaluate an expression to find Angelo's position relative to the ground.

To solve this problem, look at the meanings of words to help you:

- decide what integer starts the expression.
- decide when to add.
- decide when to subtract.

Answer each question.

1. What integer starts the expression? What word tells you if it is positive or negative?

2. When do you add? What word tells you when to add?

3. When do you subtract? What word tells you when to subtract?

4. Write and find the value of the expression to solve the problem.

5. Where is Angelo's hot air balloon in relation to the ground?

6. Is Angelo higher or lower than where he started? Explain.

LESSON
1-4

Applying Addition and Subtraction of Integers

Success for English Learners

Problem

Casey starts with \$180 in her bank account. She withdraws \$90, and then she deposits \$50. Mitchell starts with \$120 in his bank account. He deposits \$75, and then he withdraws \$45. Who has more money in the bank at the end?

Draw a diagram and evaluate.

Casey

STARTS	WITHDRAWS	DEPOSITS	
\$180	-\$90	+\$50	= \$140

Mitchell

STARTS	DEPOSITS	WITHDRAWS	
\$120	+\$75	-\$45	= \$150

Now, compare.

Casey		Mitchell
\$140	<	\$150

At the end, Mitchell has more money in his account.

1. Why do you subtract when money is withdrawn?

2. Why do you add when money is deposited?

3. Write a word problem adding and subtracting integers. Solve.

MODULE
1

Adding and Subtracting Integers

Challenge

Maria wants to compare the difficulty of different bicycle paths in her town. She recorded the elevation of the trail at each mile marker. She also calculated the difference in the elevation at each mile marker with the elevation at the previous mile marker. The difficulty score she assigned to each trail is the sum of these differences.

Trail	Elevation (ft)					
	Start	Mile 1	Mile 2	Mile 3	Mile 4	Mile 5
Easy Rider	1	-2	10	-1	120	-5
Breakneck	-2	100	-2	150	-8	250
Lake Shore	-10	0	6	55	-1	60
Mountain View	40	-2	120	35	200	180

For example, to find the difficulty of the Easy Rider trail, Maria first calculated the differences in elevation at each mile marker.

	Mile 1	Mile 2	Mile 3	Mile 4	Mile 5
Difference in Elevation	$-2 - 1 = -3$	$10 - (-2) = 12$	$-1 - 10 = -11$	$120 - (-1) = 121$	$-5 - 120 = -125$

The difficulty score of the Easy Rider is the sum of these differences.

$$-3 + 12 + (-11) + 121 + (-125) = -6$$

- Which trail has the highest difficulty rating? Show your work in a table.

Solve.

2. $-3 \square 5 \square -4 \square -10 \square 18$

Each of the boxes in the expression above can be filled with + or - .

What is the greatest possible value of the expression? Explain.

LESSON
2-1**Multiplying Integers****Practice and Problem Solving: A/B****Find each product.**

1. $4(-20)$

2. $-6(12)$

3. $(-8)(-5)$

4. $(13)(-3)$

5. $(-10)(0)$

6. $(-5)(16)$

7. $(-9)(-21)$

8. $11(-1)$

9. $18(-4)$

10. $10(8)$

11. $9(-6)$

12. $-7(-7)$

Write a mathematical expression to represent each situation. Then find the value of the expression to solve the problem.

13. You play a game where you score
- -6
- points on the first turn and on each of the next 3 turns. What is your score after those 4 turns?
-
- _____

14. The outdoor temperature declines 3 degrees each hour for 5 hours. What is the change in temperature at the end of those 5 hours?
-
- _____

15. You have \$200 in a savings account. Each week for 8 weeks, you take out \$18 for spending money. How much money is in your account at the end of 8 weeks?
-
- _____

16. The outdoor temperature was 8 degrees at midnight. The temperature declined 5 degrees during each of the next 3 hours. What was the temperature at 3 A.M.?
-
- _____

17. The price of a stock was \$325 a share. The price of the stock went down \$25 each week for 6 weeks. What was the price of that stock at the end of 6 weeks?
-
- _____

LESSON
2-1**Multiplying Integers****Practice and Problem Solving: C****Find each product.**

1. $(-14)(7)$

2. $(-24)(-5)$

3. $12(-12)$

4. $15(-9)(-1)$

5. $2(-3)(4)$

6. $-3(-6)(-2)$

7. $40(-78)(0)$

8. $-6(-60)(-4)$

9. $-24(7)(-7)$

Write a mathematical expression to represent each situation. Then find the value of the expression to solve the problem.

10. A football team loses 4 yards on each of three plays. Then they complete a pass for 9 yards. What is the change in yardage after those four plays?
- _____

11. You have \$220 in your savings account. You take \$35 from your account each week for four weeks. How much is left in your account at the end of the four weeks?
- _____

12. A submarine is at -125 feet in the ocean. The submarine makes three dives of 50 feet each. At what level is the submarine after the three dives?
- _____

Find each product. Use a pattern to complete the sentences.

13. $-1(-1)$ _____

14. $-1(-1)(-1)$ _____

15. $-1(-1)(-1)(-1)$ _____

16. $-1(-1)(-1)(-1)(-1)$ _____

17. $-1(-1)(-1)(-1)(-1)(-1)$ _____

18. When multiplying integers, if there is an odd number of negative factors, then the product is _____.

If there is an even number of negative factors, then the product is _____.

_____.

LESSON
2-1**Multiplying Integers****Practice and Problem Solving: D**

Find each product. The first one is done for you.

1. $3(-2)$

-6

2. $5(0)$

3. $(-1)(-8)$

4. $(-4)(7)$

5. $(-3)(-4)$

6. $(6)(-6)$

7. $10(-5)$

8. $-2(9)$

9. $7(-10)$

10. $-1(-1)$

11. $2(-6)$

12. $-2(-2)$

Write a mathematical expression to represent each situation. Then find the value of the expression to solve the problem. The first one is done for you.

13. You play a game where you score -3 points on the first 5 turns. What is your score after those 5 turns?

$5(-3) = -15$; -15 points

14. The outdoor temperature gets 1 degree colder each hour for 3 hours. What is the change in temperature at the end of those 3 hours?

15. A football team loses 4 yards on each of 2 plays. What is the change in yardage after those 2 plays?

16. You take \$9 out of your savings account each week for 7 weeks. At the end of 7 weeks, what is the change in the amount in your savings account?

17. The price of a stock went down \$5 each week for 5 weeks. What was the change in the price of that stock at the end of 5 weeks?

LESSON
2-1

Multiplying Integers

Reteach

You can use patterns to learn about multiplying integers.

$6(2) = 12$			
$6(1) = 6$		-6	Each product is 6 less than the previous product.
$6(0) = 0$		-6	The product of two positive integers is positive.
$6(-1) = -6$		-6	The product of a positive integer and a negative integer is negative.
$6(-2) = -12$		-6	

Here is another pattern.

$-6(2) = -12$			
$-6(1) = -6$		+6	Each product is 6 more than the previous product.
$-6(0) = 0$		+6	The product of a negative integer and a positive integer is negative.
$-6(-1) = 6$		+6	The product of two negative integers is positive.
$-6(-2) = 12$		+6	

Find each product.

1. $1(-2)$

Think: $1 \times 2 = 2$. A negative and a positive integer have a negative product.

2. $-6(-3)$

Think: $6 \times 3 = 18$. Two negative integers have a positive product.

3. $(5)(-1)$

4. $(-9)(-6)$

5. $11(4)$

Write a mathematical expression to represent each situation.

Then find the value of the expression to solve the problem.

6. You are playing a game. You start at 0. Then you score -8 points on each of 4 turns. What is your score after those 4 turns?

7. A mountaineer descends a mountain for 5 hours. On average, she climbs down 500 feet each hour. What is her change in elevation after 5 hours?

LESSON
2-1

Multiplying Integers

Reading Strategies: Use Graphic Aids

The opposite of 6 is -6 .

Losing points is the opposite of gaining points.

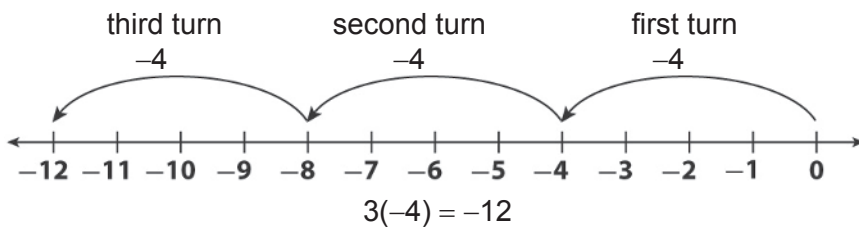
Losing 6 points is the opposite of gaining 6 points.

Answer each question.

1. What is the opposite of losing 10 points? _____

2. What is the opposite of gaining 17 points? _____

You start a game with a score of 0. You lose 4 points on each of the first three turns. How many points will you lose in all on those three turns? What will your score be after the third turn?



Use the number line to help you answer the questions.

3. Starting at zero, which direction do you move first? _____

4. How many places do you move? _____

5. Which direction do you move next? _____

6. How many places do you move? _____

7. Which direction do you move next? _____

8. How many places do you move? _____

9. How did your score change from the beginning of the game to the end of the third move?

10. What was your score at the end of the third move? _____

11. Suppose you lose another 4 points on your next move.
What would your score be at the end of that move?

LESSON
2-1

Multiplying Integers

Success for English Learners

Problem 1

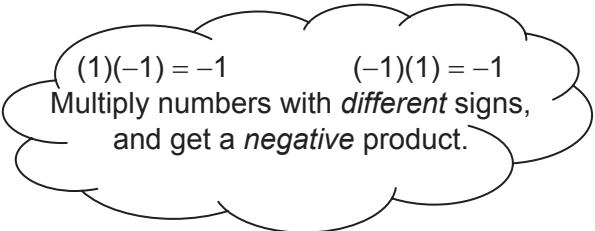
Multiply: -8×5 .

Use the absolute values of the numbers.

$|-8| = 8$ $|5| = 5$

Think about multiplying integers.

Since $8 \times 5 = 40$, $(-8)(5) = -40$



Problem 2

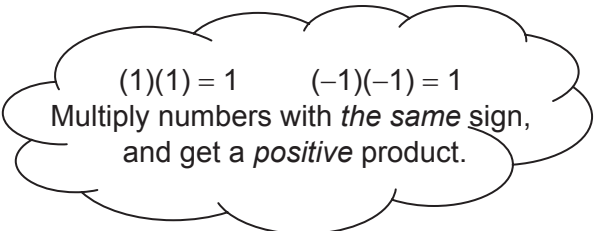
Multiply: -6×-4 .

Use the absolute values of the numbers.

$|-6| = 6$ $|-4| = 4$

Think about multiplying integers.

Since, $6 \times 4 = 24$, $(-6)(-4) = 24$



You have money in a bank account. You take out \$20 each week for three weeks. After the three weeks, what is the change in the amount of money in your account?

1. What integer shows the money you take out each week? _____
2. What integer shows the number of weeks you take money out?

3. What expression can you use to solve the problem? _____
4. What is the change in your account after three weeks? _____

Answer each question.

5. You know $50 \times 8 = 400$. Explain how that helps you find $(-50)(-8)$.

6. Are the products of 4×-8 and -4×8 the same? Explain.

LESSON

2-2

Dividing Integers**Practice and Problem Solving: A/B****Find each quotient.**

1. $7 \overline{) -84}$

2. $-38 \div -2$

3. $-27 \overline{) 81}$

4. $-28 \div 7$

5. $-121 \div -11$

6. $-35 \div 4$

Simplify.

7. $(-6 - 4) \div 2$

8. $5(-8) \div 4$

9. $-6(-2) \div 4(-3)$

Write a mathematical expression for each phrase.

10. thirty-two divided by the opposite of 4

11. the quotient of the opposite of 30 and 6, plus the opposite of 8

12. the quotient of 12 and the opposite of 3 plus the product of the opposite of 14 and 4

Solve. Show your work.

13. A high school athletic department bought 40 soccer uniforms at a cost of \$3,000. After soccer season, they returned some of the uniforms but only received \$40 per uniform. What was the difference between what they paid for each uniform and what they got for each return?

14. A commuter has \$245 in his commuter savings account.

a. This account changes by $-\$15$ each week he buys a ticket. In one time period, the account changed by $-\$240$. For how many weeks did the commuter buy tickets?

b. How much must he add to his account if he wants to have 20 weeks worth of tickets in his account?

LESSON
2-2
Dividing Integers
Practice and Problem Solving: C
Simplify.

1. $-\frac{-8}{-2} + (-12)$

2. $\frac{6}{-3} - \frac{15-7}{-2}$

3. $3 - 2(4 - 7) \div 9$

The integers from -3 to $+3$ can be used in the blanks below. Which of these integers produces a positive, even integer for the expression?

Show your work for those that do.

4. $-\frac{8}{2} + 4(\text{_____}) - 2$

5. $\frac{(\text{_____})}{4} + \frac{3}{2}$

6. $\text{_____} \div \frac{2}{-3}$

7. $\left(\frac{-1}{\text{_____}}\right) \div -\frac{1}{2}$

Solve. Show your work.

8. In a sports competition, Alyssa was penalized -16 points. She received the same number of penalty points in each of 4 events. How many points was she penalized in each event?

9. The surface temperature of a deep, spring-fed lake is 70°F . The lake temperature drops 2°F for each yard below the lake surface until a depth of 6 yards is reached. From 6 yards to 15 yards deep, the temperature is constant. From 15 yards down to the spring source, the temperature *increases* 3°F per *foot* until the spring source is reached at 20 yards below the surface.

- a. What is the temperature at 10 yards below the surface?

- b. What is the temperature at 50 feet below the surface?

- c. Write an expression for finding the lake temperature at the spring source.

LESSON
2-2

Dividing Integers

Practice and Problem Solving: D

Find the quotient. The first one is done for you.

1. $-3 \overline{) -15}$

5

2. $27 \div (-3)$

3. $\frac{28}{-7}$

Compare the quotients. Write $>$, $<$, or $=$.

4. $-4 \overline{) -16}$ $-16 \overline{) -4}$

5. $11 \div 77$ $77 \div 11$

6. $\frac{48}{-6}$ $\frac{-48}{6}$

Write a mathematical expression for the written expression. Then solve. The first one is done for you.

7. the opposite of 45 divided by 5

$-45 \div 5 = -9$

8. fifty-five over negative eleven

9. negative 38 divided by positive 19

10. negative four divided by negative two

Solve. Show your work. The first one is done for you.

11. Four investors lost 24 percent of their combined investment in a company. On average, how much did each investor lose?

$-24 \div 4 = -6$; On average, each investor lost 6%.

12. The temperature in a potter's kiln dropped 760 degrees in 4 hours. On average, how much did the temperature drop per hour?

13. The value of a car decreased by \$5,100 over 3 years. On average, how much did its value decrease each year?

LESSON
2-2

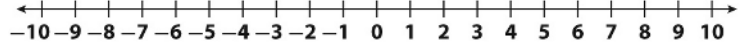
Dividing Integers

Reteach

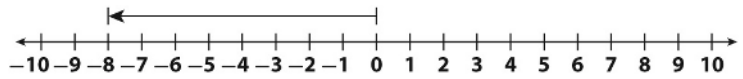
You can use a number line to divide a negative integer by a positive integer.

$$-8 \div 4$$

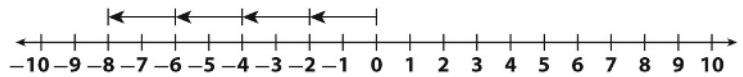
Step 1 Draw the number line.



Step 2 Draw an arrow to the left from 0 to the value of the dividend, -8 .



Step 3 Divide the arrow into the same number of small parts as the divisor, 4.



Step 4 How long is each small arrow? When a negative is divided by a positive the quotient is negative, so the sign is negative.

← Each arrow is -2 .

So, $-8 \div 4 = -2$.

On a number line, in which direction will an arrow that represents the dividend point? What is the sign of the divisor? Of the quotient?

1. $54 \div (-9)$

2. $-4 \overline{) -52}$

3. $\frac{-39}{3}$

Dividend: _____

Dividend: _____

Dividend: _____

Sign of

Sign of

Sign of

Divisor: _____

Divisor: _____

Divisor: _____

Sign of

Sign of

Sign of

Quotient: _____

Quotient: _____

Quotient: _____

Complete the table.

4.

Divisor	Dividend	Quotient
+	+	
	+	
	-	-
		+

LESSON
2-2**Dividing Integers****Reading Strategies: Understand Symbols**

Different symbols can be used to show division with integers. Each symbol can also be described with words.

$-8 \overline{) -136}$ ←	The <i>dividend</i> is -136 . The <i>divisor</i> is -8 . “ -8 goes into -136 .”
$63 \div (-3)$ ←	The <i>dividend</i> is 63 . The <i>divisor</i> is -3 . “ 63 divided by -3 .”
$\frac{-125}{5}$ ←	The <i>dividend</i> is -125 . The <i>divisor</i> is 5 . “ -125 over 5 .”

Name the dividend and the divisor. Then find the quotient.

1. The total distance is 3,600 kilometers. The average speed is 225 kilometers per hour. How long did it take to drive that distance?

dividend: _____ divisor: _____ quotient: _____

2. The temperature dropped 35 degrees in 7 hours. What was the average drop in temperature per hour?

dividend: _____ divisor: _____ quotient: _____

3. A beverage company produced 1,600 liters of fruit punch, which will be bottled in 2-liter bottles. How many bottles can be filled?

dividend: _____ divisor: _____ quotient: _____

Use words to describe each division problem two ways.

4. $-6 \overline{) 102}$ _____

5. $\frac{-221}{-17}$ _____

LESSON
2-2

Dividing Integers

Success for English Learners

Problem 1

Words help you find the dividend, the divisor, and the quotient. They also tell the sign of the numbers.

“45 divided by negative 5 is the opposite of 9.”

$\underbrace{\hspace{1.5cm}}$ $\underbrace{\hspace{1.5cm}}$ $\underbrace{\hspace{1.5cm}}$
 dividend divisor quotient

→ $-5 \overline{)45} = -9$

OR $\frac{45}{-5} = -9$

OR $45 \div (-5) = -9$

Write a math expression for the words. Then solve.

1. the opposite of 210 over 70

2. negative 4,200 divided by 300

3. negative 50 divided by positive 10

4. 54 divided by 27

Problem 2

Find the sign of the quotient before dividing.

$\frac{-720}{-8}$ ←

Negative (–) divided by negative (–) gives positive (+).

Negative (–) divided by positive (+) gives negative (–).

Positive (+) divided by negative (–) gives negative (–).

Positive (+) divided by positive (+) gives positive (+).

Write the sign of the quotient, + or –. Then find the quotient.

5. $\frac{33}{33}$

6. $\frac{128}{-4}$

7. $\frac{-100}{25}$

8. $\frac{-75}{-15}$

Sign: _____

Sign: _____

Sign: _____

Sign: _____

Quotient:

Quotient:

Quotient:

Quotient:

LESSON
2-3**Applying Integer Operations****Practice and Problem Solving: A/B****Find the value of each expression.**

1. $(-3)(-2) + 8$

2. $(-18) \div 3 + (5)(-2)$

3. $7(-3) - 6$

4. $24 \div (-6)(-2) + 7$

5. $4(-8) + 3$

6. $(-9)(0) + (8)(-5)$

Compare. Write $<$, $=$, or $>$.

7. $(-5)(8) + 3$ $(-6)(7) + 1$

8. $(-8)(-4) + 16 \div (-4)$ $(-9)(-3) + 15 \div (-3)$

Write an expression to represent each situation. Then find the value of the expression to solve the problem.

9. Dave owns 15 shares of ABC Mining stock. On Monday, the value of each share rose \$2, but on Tuesday the value fell \$5. What is the change in the value of Dave's shares?
- _____

10. To travel the Erie Canal, a boat must go through locks that raise or lower the boat. Traveling east, a boat would have to be lowered 12 feet at Amsterdam, 11 feet at Tribes Hill, and 8 feet at Randall. By how much does the elevation of the boat change between Amsterdam and Randall?
- _____

11. The Gazelle football team made 5 plays in a row where they gained 3 yards on each play. Then they had 2 plays in a row where they lost 12 yards on each play. What is the total change in their position from where they started?
- _____

12. On Saturday, Mrs. Armour bought 7 pairs of socks for \$3 each, and a sweater for her dog for \$12. Then she found a \$5 bill on the sidewalk. Over the course of Saturday, what was the change in the amount of money Mrs. Armour had?
- _____

LESSON
2-3**Applying Integer Operations****Practice and Problem Solving: C**

Complete the table to answer 1–4.

	You Own	Company	Monday	Tuesday	Wednesday	Net Gain or Loss
1.	5 shares	ABC	−\$2	+\$5	−\$1	
2.	2 shares	DEF	+\$8	−\$7	−\$10	
3.	8 shares	GHI	−\$2	+\$9	+\$6	
4.	7 shares	JKL	+\$5	−\$12	+\$3	

5. What expression shows your net gain or loss on GHI Company?

6. How much value did you gain or lose overall? _____

Write an expression to represent each situation. Then, find the value of the expression to solve the problem.

7. A submarine cruised below the surface of the water. During a training exercise, it made 4 dives, each time descending 45 feet more. Then it rose 112 feet. What is the change in the submarine's position?

8. A teacher wanted to prevent students from guessing answers on a multiple-choice test. The teacher graded 5 points for a correct answer, 0 points for no answer, and −2 points for a wrong answer. Giselle answered 17 questions correctly, left 3 blank, and had 5 wrong answers. She also got 8 out of 10 possible points for extra credit. What was her final score?

9. Hugh wrote six checks from his account in the following amounts: \$20, \$20, \$12, \$20, \$12, and \$42. He also made a deposit of \$57 and was charged a \$15 service fee by the bank. What is the change in Hugh's account balance?

10. a. Without finding the product, what is the sign of this product? Explain how you know.

$$(-4)(-1)(-2)(-6)(-3)(-5)(-2)(-2)$$

b. Find the product. _____

LESSON
2-3**Applying Integer Operations****Practice and Problem Solving: D**

Find the value of each expression. Show your work. The first one is done for you.

1. $15 + (-6)(2)$

$$= 15 + (-12) \quad \text{Multiply}$$

$$= 3 \quad \text{Add.}$$

2. $(-5)(-3) + 18$

3. $42 \div (-6) + 23$

4. $52 + 45 \div (-9)$

Write an expression to represent each situation. Then find the value of the expression to solve the problem. The first one is done for you.

5. Mr. Carlisle paid his utility bills last weekend. He paid \$50 to the phone company, \$112 to the power company, and \$46 to the water company. After he paid those bills, what was the change in the total amount of money that Mr. Carlisle had?

$$(-50) + (-112) + (-46) = -208; \text{ He had } \$208 \text{ less.}$$

6. Over 5 straight plays, a football team gained 8 yards, lost 4 yards, gained 7 yards, gained 3 yards, and lost 11 yards. What is the team's position now compared to their starting position?

7. At the grocery store, Mrs. Knight bought 4 pounds of apples for \$2 per pound and 2 heads of lettuce for \$1 each. She had a coupon for \$3 off the price of the apples. After her purchases, what was the change in the amount of money that Mrs. Knight had?

8. The depth of the water in a water tank changes every time someone in the Harrison family takes a bath or does laundry. A bath lowers the water level by 4 inches. Washing a load of laundry lowers the level by 2 inches. On Monday the Harrisons took 3 baths and washed 4 loads of laundry. By how much did the water level in the water tank change?

LESSON
2-3**Applying Integer Operations****Reteach**

To evaluate an expression, follow the order of operations.

1. Multiply and divide in order from left to right. $(-5)(6) + 3 + (-20) \div 4 + 12$
 $-30 + 3 + (-20) \div 4 + 12$

$$-30 + 3 + (-20) \div 4 + 12$$

$$-30 + 3 + (-5) + 12$$

2. Add and subtract in order from left to right. $-30 + 3 + (-5) + 12$
 $-27 + (-5) + 12$
 $-32 + 12 = -20$

Name the operation you would do first.

1. $-4 + (3)(-8) + 7$

2. $-3 + (-8) - 6$

3. $16 + 72 \div (-8) + 6(-2)$

4. $17 + 8 + (-16) - 34$

5. $-8 + 13 + (-24) + 6(-4)$

6. $12 \div (-3) + 7(-7)$

7. $(-5)6 + (-12) - 6(9)$

8. $14 - (-9) - 6 - 5$

Find the value of each expression.

9. $(-6) + 5(-2) + 15$

10. $(-8) + (-19) - 4$

11. $3 + 28 \div (-7) + 5(-6)$

12. $15 + 32 + (-8) - 6$

13. $(-5) + 22 + (-7) + 8(-9)$

14. $21 \div (-7) + 5(-9)$

LESSON
2-3

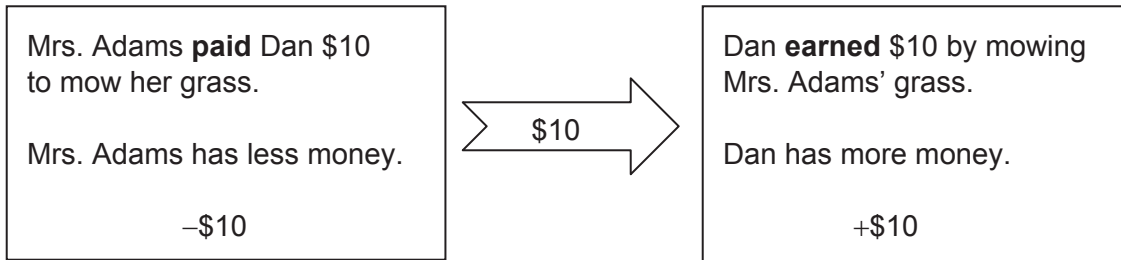
Applying Integer Operations

Reading Strategies: Use Context

Someone mentions an amount when describing a mathematical situation.

Should you represent that number with a positive integer or a negative integer?

Use key words to help you decide.



Words That Show Negative Numbers	Words That Show Positive Numbers
He spent \$20. -20	He found \$5. +5
They lost 8 yards. -8	120 feet above sea level. +120
15° below zero. -15	They gained 38 yards. +38

Write an expression to represent each situation. Underline the words you used to decide whether each number in your expression should be positive or negative. Find the value of the expression to solve the problem.

1. Antoine went to the store. He paid \$3 each for 4 pounds of grapefruit. When he got home, his mother gave him \$7 for some of the grapefruit. What is the change in the amount of money Antoine has?

2. Matt went scuba diving. He dove to a depth of 48 feet below the surface. Elena dove one-fourth as deep as Matt. What number describes the depth Elena dove?

3. The Cougars football team had 3 straight plays where they lost 5 yards on each play. On the fourth play they gained 32 yards. How many yards did they gain or lose for those 4 plays?

LESSON
2-3

Applying Integer Operations

Success for English Learners

Problem 1

What is the value?

$$(-3)(2) + 4 + (-16) \div 4 + 3$$

1. First, **multiply and divide**

from left to right.

$$(-3)(2) + 4 + (-16) \div 4 + 3$$

$$-6 + 4 + (-4) + 3$$

2. Then **add and subtract**

from left to right.

$$-6 + 4 + (-4) + 3$$

$$-2 + (-4) + 3$$

$$-6 + 3 = -3$$

Problem 2

To solve the problem, write a math expression.

Manny bought 8 gallons of gas. He paid \$4 for each gallon.

What is the change in the amount of money Manny has?

Manny *paid* \$4 for each gallon, so the 4 is negative.

Manny bought 8 gallons. So, multiply by the cost of one gallon by 8.

$$8(-4) = -32$$

Manny has \$32 less now.

Find the value.

1. $(-4)(-5) + 19$

2. $(-9)(4) + 31$

3. $(-36) \div 9 + 4 + (-2)(-3)$

4. a. Write a problem that could be shown by this expression.

$$3(-20) + 5$$

- b. Find the value of the expression to solve the problem.

MODULE
2

Multiplying and Dividing Integers

Challenge

- Write an expression with integers that uses all four operations, includes at least 5 terms, and that, when simplified, is -17 . Use the rules for the order of operations. Show your work.

- You have two sets of integer cards -15 to 15 , and four sets of operation cards ($+$, $-$, \times , and \div). Make up a game that could be played using these cards. Write the rules for your game.

- Explain to a new student how to simplify the expression below. Show the updated expression after each step you take.

$$(-8) + (-3) + (-4)(7) \div 14 + 9(-2)$$

LESSON
3-1
Rational Numbers and Decimals
Practice and Problem Solving: A/B

Write each rational number as a terminating decimal.

1. $\frac{19}{20}$

2. $-\frac{1}{8}$

3. $\frac{17}{5}$

Write each rational number as a repeating decimal.

4. $-\frac{7}{9}$

5. $\frac{11}{15}$

6. $\frac{8}{3}$

Write each mixed number as an improper fraction and as a decimal.
Then tell whether the decimal is terminating or repeating.

7. $3\frac{2}{9}$ _____

8. $15\frac{1}{20}$ _____

9. $-5\frac{3}{10}$ _____

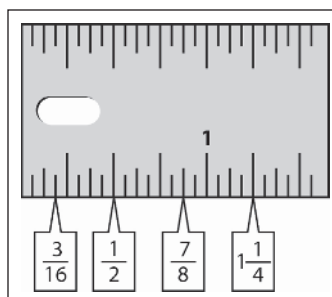
10. In part a and in part b, use each of the digits 2, 3, and 4 exactly once.

a. Write a mixed number that has a terminating decimal, and write the decimal.

b. Write a mixed number that has a repeating decimal, and write the decimal.

11. The ruler is marked at every $\frac{1}{16}$ inch. Do the labeled measurements

convert to repeating or terminating decimals? _____



LESSON
3-1

Rational Numbers and Decimals

Practice and Problem Solving: C

Use the table to answer problems 1 to 4.

The table lists the commuting times for 5 people. Write each ratio in the form $\frac{a}{b}$, and then as a decimal. Tell whether each decimal is a terminating or a repeating decimal.

1. Beau's time to the sum of Charra's and DeLee's times

2. The product of DeLee's time and Beau's time to Adelle's time

3. Write two or three rational numbers with Emmet's time in each denominator. Is the rational number always a terminating decimal? Justify your answer.

4. Use Adelle's time as the denominator of rational numbers. Find a numerator that results in a terminating decimal and find a numerator that results in a repeating decimal. Explain how to find numerators for each type of rational number.

5. Is $\frac{1.5}{7.5}$ a rational number? If not, explain why. If so, explain why and write it as a decimal.

Person	Commuting Time (min.)
Adelle	15
Beau	25
Charra	10
DeLee	8
Emmet	20

LESSON
3-1

Rational Numbers and Decimals

Practice and Problem Solving: D

Write each rational number as a repeating decimal or a terminating decimal. Then tell whether the decimal is terminating or repeating. The first two are done for you.

1. $\frac{13}{20}$ **0.65, terminating** _____

$$\begin{array}{r} 0.65 \\ 20 \overline{)13.00} \\ \underline{120} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

2. $4\frac{2}{3}$ _____ **$4.\overline{6}$, repeating**

$$4\frac{2}{3} = \frac{12}{3} + \frac{2}{3} = \frac{14}{3}$$

$$\begin{array}{r} 4.666 \\ 3 \overline{)14.000} \\ \underline{12} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \end{array}$$

3. $\frac{5}{9}$ _____

4. $3\frac{5}{6}$ _____

5. $8\frac{3}{4}$ _____

6. $10\frac{5}{8}$ _____

The decimal for $\frac{5}{16}$ is 0.3125. Use that value to write each decimal.

7. $1\frac{5}{16}$

8. $7\frac{5}{16}$

9. $26\frac{5}{16}$

The decimal for $\frac{4}{15}$ is 0.266... or $0.2\overline{6}$. Use that value to write each decimal.

10. $1\frac{4}{15}$

11. $17\frac{4}{15}$

12. $23\frac{4}{15}$

LESSON

3-1

Rational Numbers and Decimals**Reteach**

A teacher overheard two students talking about how to write a mixed number as a decimal.

Student 1: I know that $\frac{1}{2}$ is always 0.5, so $6\frac{1}{2}$ is 6.5 and $11\frac{1}{2}$ is 11.5.

I can rewrite any mixed number if the fraction part is $\frac{1}{2}$.

Student 2: You just gave me an idea to separate the whole number part and the fraction part. For $5\frac{1}{3}$, the fraction part is

$\frac{1}{3} = 0.333\dots$ or $0.\bar{3}$, so $5\frac{1}{3}$ is $5.333\dots$ or $5.\bar{3}$.

I can always find a decimal for the fraction part, and then write the decimal next to the whole number part.

The teacher asked the two students to share their ideas with the class.

For each mixed number, find the decimal for the fraction part. Then write the mixed number as a decimal.

1. $7\frac{3}{4}$

2. $11\frac{5}{6}$

3. $12\frac{3}{10}$

4. $8\frac{5}{18}$

For each mixed number, use two methods to write it as a decimal. Do you get the same result using each method?

5. $9\frac{2}{9}$

6. $21\frac{5}{8}$

LESSON
3-1

Rational Numbers and Decimals

Reading Strategies: Understanding Key Phrases

The definition of *rational number* uses two key phrases.

- A rational number is a number that can be written as a **ratio of two integers** a and b ...
- A rational number is a number that **can be** written as a ratio of two integers a and b ...

The phrase **ratio of two integers** means fractions such as $\frac{5}{8}$, $\frac{9}{11}$, or $\frac{-7}{12}$.

Each fraction is a ratio and both the numerator and the denominator in the ratio are integers.

Tell whether or not each numerator and denominator is an integer.

1. $\frac{-3}{5}$ _____

2. $\frac{2}{1.17}$ _____

3. $\frac{1}{\frac{1}{3}}$ _____

4. $\frac{\sqrt{2}}{\sqrt{4}}$ _____

The phrase **can be** means that you have to think about rewriting the fraction.

For example, question 2 shows the fraction $\frac{2}{1.17}$. We can rewrite $\frac{2}{1.17}$ as $\frac{200}{117}$, so $\frac{2}{1.17}$ is a rational number because it *can be* written as a ratio of two integers.

Similarly, $\frac{1}{\frac{1}{3}} = 3$ or $\frac{3}{1}$, so it is a rational number because it *can be* written

as a ratio of two integers.

Explain whether or not each fraction *can be* written as a ratio of two integers.

5. $\frac{\sqrt{3}}{\sqrt{6}}$ _____

6. $\frac{\sqrt{2}}{\sqrt{2}}$ _____

7. $\frac{\sqrt{4}}{\sqrt{25}}$ _____

8. $\frac{\sqrt{1}}{2}$ _____

LESSON
3-1

Rational Numbers and Decimals

Success for English Learners

Problem 1

How many digits repeat?

$$5\frac{2}{3} = 5.\underline{6}\underline{6}\underline{6}\dots$$

One digit repeats.

$$\frac{7}{33} = 0.\underline{2}\underline{1}\underline{2}\underline{1}\underline{2}\dots$$

Two digits repeat.

$$\frac{11}{333} = 0.\underline{0}\underline{3}\underline{3}\underline{0}\underline{3}\underline{3}\underline{0}\underline{3}\underline{3}\dots$$

Three digits repeat.

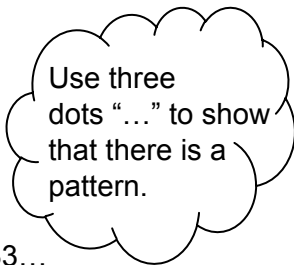
Problem 2

How do you show a repeating decimal?

$$5\frac{2}{3} = 5.666\dots$$

$$\frac{7}{33} = 0.212121\dots$$

$$\frac{11}{333} = 0.033033033\dots$$



or

$$5\frac{2}{3} = 5.\overline{6}$$

$$\frac{7}{33} = 0.\overline{21}$$

$$\frac{11}{333} = 0.\overline{033}$$



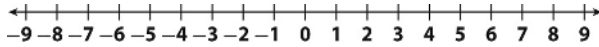
1. The decimal for $\frac{5,141}{9,999}$ is 0.51415141... .

How many repeating digits does it have?

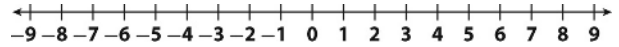
- A It has one repeating digit, which is 5.
 - B It has two repeating digits, 5 and 1.
 - C It has a group of three repeating digits, 141.
 - D It has a group of four repeating digits, 5,141.
2. How can you use a bar to show that 0.51415141... is a repeating decimal?
- A $0.\overline{5141}$
 - B $0.5\overline{141}$
 - C $0.\overline{51415}$
 - D $0.5\overline{14\overline{1}}$
3. Use three dots “...” to write a decimal that has two repeating digits. Then use a bar to write the same repeating decimal.

LESSON
3-2**Adding Rational Numbers****Practice and Problem Solving: A/B****Use a number line to find each sum.**

1. $-3 + 4$



2. $1 + (-8)$



Find each sum without using a number line.

3. $4 + 5$

4. $-3 + \frac{1}{2}$

5. $-\frac{2}{9} + \frac{3}{9}$

6. $-3.5 + (-4.9)$

7. $-2\frac{1}{4} + \left(-3\frac{1}{4}\right)$

8. $-0.6 + (-2.5)$

9. $-\frac{3}{4} + \frac{1}{5}$

10. $3 + (-7.5) + 1.2$

11. $-1.32 + 5.02 + (-1.24)$

12. $-3 + (-1.35) + 2.5$

13. $-6.5 + (-0.15) + (-0.2)$

14. $-\frac{3}{2} - \frac{7}{4} + \frac{1}{8}$

Solve.

15. Alex borrowed \$12.50 from his friend Danilo. He paid him back \$8.75. How much does he still owe?

16. A football team gains 18 yards in one play and then loses 12 yards in the next. What is the team's total yardage?

17. Dee Dee bought an apple for \$0.85, a sandwich for \$4.50, and a bottle of water for \$1.50. How much did Dee Dee spend?

18. Andre went hiking near his house. The first trail he hiked on took him 4.5 miles away from his house. The second trail he hiked took him 2.4 miles closer to his house. The third trail took him 1.7 miles further away from his house. After Andre hiked the three trails, how far from his house was he?

LESSON

3-2

Adding Rational Numbers**Practice and Problem Solving: C****Find each sum.**

1. $3\frac{1}{4} + \left(-1\frac{1}{2}\right) + 2\frac{1}{4}$

2. $3\frac{3}{5} + \left(-1\frac{4}{9}\right) + \frac{1}{5}$

3. $-\frac{1}{9} + 8\frac{3}{5} - \frac{1}{15}$

4. $-3.5 + (-4.9) + 0.8$

5. $-\frac{1}{12} + \left(-3\frac{3}{8}\right) + \frac{4}{3}$

6. $-0.25 + (-1.65) + 0.77$

7. $-\frac{3}{4} + \frac{1}{5} + \frac{2}{9}$

8. $0.3 + (-5.5) + 5.2$

9. $-1.091 + 12.12 + (-1.1)$

10. $-3.24 + (-1.55) + 2.512$

11. $-1.27 + (-0.35) + (-0.13)$

12. $-1\frac{1}{2} + \left(-2\frac{12}{23}\right) + 5\frac{7}{46}$

Solve.

13. Marley bought an action figure for \$10.99, a board game for \$24.95 and a book for \$5.99. She paid with a \$50 bill. How much change did Marley receive?

14. Tim bought a pen for \$2.25, a pencil for \$0.59, a notebook for \$6.49, and a highlighter for \$1.49. He used a coupon that gave him \$5.25 off his entire purchase. How much did he spend in total?

15. During the first hour of a snowstorm, $1\frac{1}{2}$ inches of snow fell. In the next hour, $4\frac{3}{8}$ inches fell. In the third hour, the snow stopped and $\frac{7}{8}$ inches of snow melted. How much snow was on the ground at the end of the third hour?

LESSON
3-2

Adding Rational Numbers

Practice and Problem Solving: D

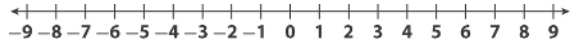
Use a number line to find each sum. The first one is done for you.

1. $2 + 3$

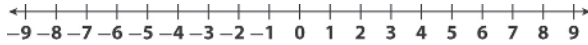


5

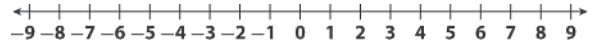
2. $-2 + 1$



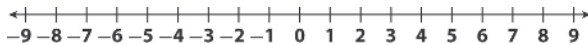
3. $-5 + (-3)$



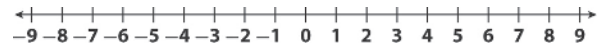
4. $-2 + 5$



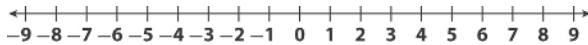
5. $1 + 7$



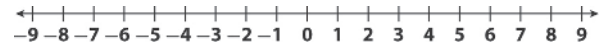
6. $-8 + 2$



7. $1 + (-0.5)$



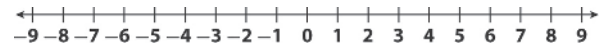
8. $-2.5 + 0.5$



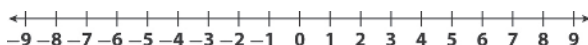
9. $0.5 + 1.5$



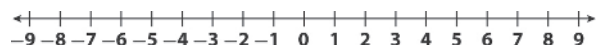
10. $-1 + \frac{1}{2}$



11. $\frac{1}{2} + 1\frac{1}{2}$



12. $-\frac{1}{2} + \frac{1}{2}$



Find each sum without using a number line. The first one is done for you.

13. $5 + (-1)$

4

14. $\frac{2}{3} + \frac{5}{3}$

15. $-\frac{1}{8} + \frac{7}{8}$

16. $-1.5 + (-1.9)$

17. $-2 + (-1.2)$

18. $-4.0 + 3.5$

19. $-1\frac{3}{4} + \frac{1}{4}$

20. $-7 + 4$

21. $-0.1 + (-0.8)$

LESSON
3-2

Adding Rational Numbers

Reteach

This balance scale “weighs” positive and negative numbers. Negative numbers go on the left of the balance. Positive numbers go on the right.



The scale will tip to the left side because the sum of -11 and $+8$ is negative.

$$-11 + 8 = -3$$



The scale will tip to the right side because the sum of $-2\frac{1}{2}$ and $+7$ is positive.

$$-2\frac{1}{2} + 7 = +4\frac{1}{2}$$



Both -0.2 and -1.5 go on the left side. The scale will tip to the left side because the sum of -0.2 and -1.5 is negative.

$$-0.2 + (-1.5) = -1.7$$

Find $3 + (-9)$.

Should you add or subtract?

Will the sum be positive or negative?

$$3 + (-9) = -6$$

$$|9| - |3|$$

the sign of the integer with the greatest absolute value

Find each sum.

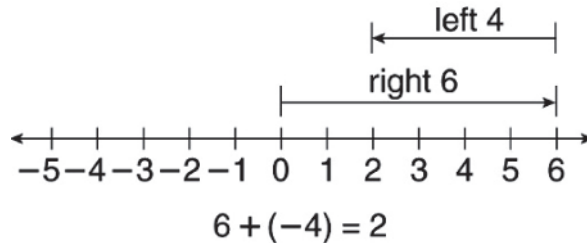
1. $-2 + 4 =$ _____ 2. $3 + (-8) =$ _____ 3. $-5 + (-2) =$ _____

4. $2.4 + (-1.8) =$ _____ 5. $1.1 + 3.6 =$ _____ 6. $-2.1 + (-3.9) =$ _____

7. $\frac{4}{5} + \left(-\frac{1}{5}\right) =$ _____ 8. $-1\frac{1}{3} + \left(-\frac{1}{3}\right) =$ _____ 9. $-\frac{7}{8} + \frac{3}{8} =$ _____

LESSON
3-2**Adding Rational Numbers****Reading Strategies: Use Graphic Aids**

Randy's football team had the ball on the zero yard line. On their first play they gained 6 yards. On the second play they lost 4 yards. On what yard line is the ball now?

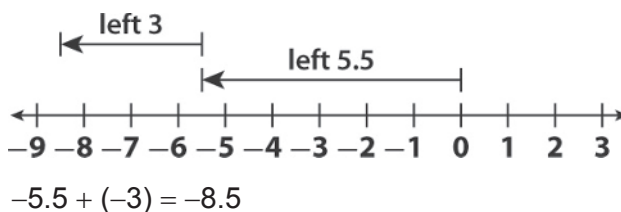


Use the number line to help you answer the questions.

1. On which number do you begin? _____
2. Which direction do you move first? How many places do you move?

3. Which direction do you move next? How many places do you move?

When Angela went to bed, the temperature was zero degrees. Two hours later, the temperature had gone down 5.5 degrees. By the time Angela got up the temperature had gone down another 3 degrees. What was the temperature when she got up?



Use the number line to help you answer the questions.

4. On which number do you begin? _____
5. In which direction do you move first? How many spaces?

6. In which direction do you move next? How many spaces?

LESSON
3-2

Adding Rational Numbers

Success for English Learners

Problem 1

Add \$3.62 and \$18.57.

$$\begin{array}{r} 3 \bullet 62 \\ +18 \bullet 57 \\ \hline 22 \bullet 19 \end{array}$$

Line up the decimal points.

Does this make sense?.

Check

3 dollars



2 quarters



1 dime



2 pennies



rounds to \$4

18 dollars



2 quarters



1 nickel



2 pennies

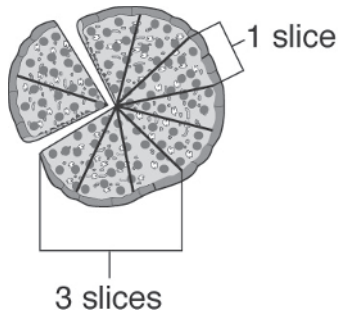


rounds to \$19

$4 + 19 = 23$, so \$22.19 is reasonable.

Problem 2

Add $\frac{3}{10} + \frac{1}{10}$.



$3 + 1 = 4$ slices

$\frac{3}{10}$ of the pizza + $\frac{1}{10}$ of the pizza = $\frac{3}{10} + \frac{1}{10} = \frac{4}{10}$ of the pizza.

1. Why is it important to line up decimals when adding decimal numbers?

2. Explain what the denominator represents in Problem 2.

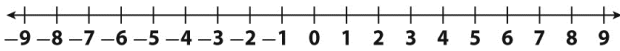
LESSON
3-3

Subtracting Rational Numbers

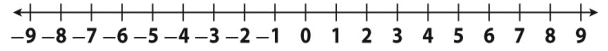
Practice and Problem Solving: A/B

Use a number line to find each difference.

1. $-5 - 4$



2. $1 - (-8)$



Find each difference without using a number line.

3. $4 - (-5)$

4. $-5 - \frac{1}{2}$

5. $\frac{1}{7} - \frac{3}{7}$

6. $-3.7 - (-4.9)$

7. $-2\frac{1}{4} - (-3)$

8. $-1.6 - 2.1$

9. $-4\frac{3}{4} - \frac{3}{4}$

10. $2 - (-7.5) - 1.2$

11. $-0.02 - 9.02 - 0.04$

12. $4 - (-0.25) - 0.5$

13. $-5.1 - (-0.1) - 1.2$

14. $-\frac{3}{5} - \frac{7}{5} - \left(-\frac{2}{5}\right)$

Solve.

15. The temperature on Monday was -1.5°C . The temperature on Tuesday was 2.6°C less than the temperature on Monday. What was the temperature on Tuesday?

16. A diver dove to a location $6\frac{3}{5}$ meters below sea level. He then dove to a second location $8\frac{1}{5}$ meters below sea level. How many meters are there between the two locations?

LESSON
3-3**Subtracting Rational Numbers****Practice and Problem Solving: C****Find each difference.**

1. $-3\frac{1}{3} - 5\frac{2}{3} - \left(-2\frac{1}{3}\right)$

2. $3\frac{3}{7} - 1\frac{2}{3} - \frac{5}{7}$

3. $-\frac{1}{10} - 4\frac{3}{5} - 5\frac{3}{10}$

4. $-1.5 - 4.9 - 0.8$

5. $-\frac{1}{12} - 3\frac{3}{8} - \left(-\frac{4}{3}\right)$

6. $-9.54 - 1.651 - 0.988$

7. $-\frac{5}{6} - \frac{17}{18} - \left(-\frac{2}{9}\right)$

8. $-0.03 - (-5.51) - 5.12$

9. $-1.099 - 12.001 - 0.09$

10. $-1.02 - 1.99 - 1.34$

11. $-1.65 - (-0.45) - (-0.15)$

12. $-3\frac{1}{2} - \left(-5\frac{5}{9}\right) - 9\frac{1}{18}$

Solve.

13. If
- x
- equals
- $8 - (-2.25)$
- and
- y
- equals
- $6 - 4.2 - (-4.9)$
- , what is the value of
- $x - y$
- ?
-
- _____

14. Alex's score for a game is given by the expression
- $9 - 8.2 - (-1.9)$
- . Beth's score for the same game is
- $-8 - (-5.4) - 1.8$
- . Whose score was higher? By how much?
-
- _____

15. The temperature on Monday was
- 14°C
- . On Tuesday the temperature decreased by
- 5°
- . On Wednesday the temperature decreased another
- 10°
- . On Thursday the temperature increased by
- 8°
- . What was the temperature at the end of the day on Thursday?
-
- _____

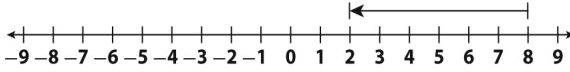
LESSON
3-3

Subtracting Rational Numbers

Practice and Problem Solving: D

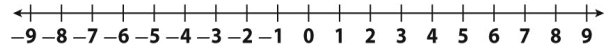
Use a number line to find each difference. The first one is done for you.

1. $8 - 6$

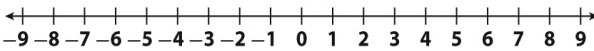


2

2. $5 - (-1)$

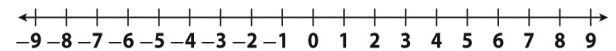


3. $-5 - (-2)$

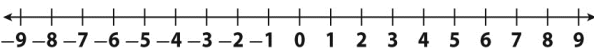


-3

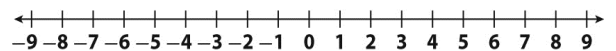
4. $-2 - 5$



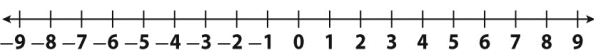
5. $1 - 4$



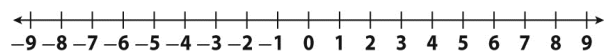
6. $4 - (-4)$



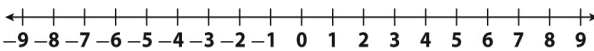
7. $1 - (-0.5)$



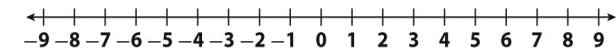
8. $-2 - 1$



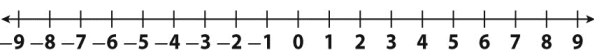
9. $1.5 - 3$



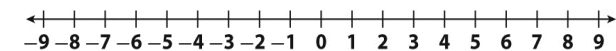
10. $2 - \frac{1}{2}$



11. $\frac{1}{2} - 1\frac{1}{2}$



12. $-\frac{1}{2} - 1$



Find each difference without using a number line. The first one is done for you.

13. $6 - (-1)$

7

14. $\frac{1}{3} - \frac{5}{3}$

15. $-\frac{1}{8} - \frac{3}{8}$

16. $-1.5 - (-2.9)$

17. $-2 - 0.2$

18. $-4.0 - 3.8$

19. $-1\frac{3}{4} - \frac{1}{4}$

20. $-2 - 4.5$

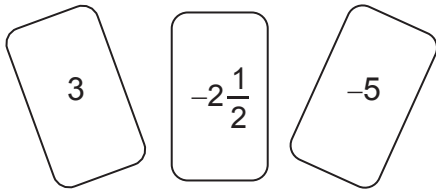
21. $-0.2 - 0.8$

LESSON
3-3

Subtracting Rational Numbers

Reteach

The total value of the three cards shown is $-4\frac{1}{2}$.



What if you **take away** the $-2\frac{1}{2}$ card?

Cards 3 and -5 are left.
Their sum is -2.

$$\text{So, } -4\frac{1}{2} - \left(-2\frac{1}{2}\right) = -2.$$

What if you **take away** the -5 card?

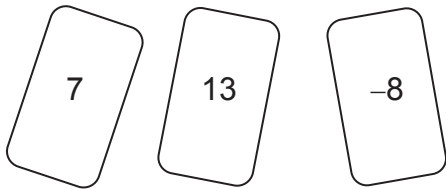
Cards 3 and $-2\frac{1}{2}$ are left.

Their sum is $\frac{1}{2}$.

$$\text{So, } -4\frac{1}{2} - (-5) = \frac{1}{2}$$

Answer each question.

1. The total value of the three cards shown is 12.



a. What is the value if you take away just the 7? _____

b. What is the value if you take away just the 13? _____

c. What is the value if you take away just the -8? _____

2. Subtract $-4 - (-2)$.

a. $-4 < -2$. So the answer will be a _____ number.

b. $|4| - |2| =$ _____

c. $-4 - (-2) =$ _____

Subtract.

3. $31 - (-9) =$ _____

4. $15 - 18 =$ _____

5. $-9 - 17 =$ _____

6. $2.6 - (-1.6) =$ _____

7. $4.5 - 2.5 =$ _____

8. $-2.0 - 1.25 =$ _____

9. $\frac{4}{5} - \left(-\frac{1}{5}\right) =$ _____

10. $-2\frac{1}{3} - \left(-\frac{1}{3}\right) =$ _____

11. $-\frac{7}{8} - \frac{3}{8} =$ _____

LESSON
3-3

Subtracting Rational Numbers

Reading Strategies: Use a Graphic Organizer

A graphic organizer is useful for subtracting decimals.

Find 19.2 – 7.54.

Step 1: Draw a table that has three rows. Include enough columns for each place that has a digit in either of the numbers. Include a separate column for the decimal points.

Step 2: Write one digit in each square. Carefully line up the decimals.

1	9	.	2	
–	7	.	5	4

Step 3: If there is a square without a number, insert a zero as a placeholder.

1	9	.	2	0
–	7	.	5	4

Step 4: Subtract each column to find the answer.

1	9	.	2	0
–	7	.	5	4
1	1	.	6	6

1. How do you place the numbers in the table?

2. Why was a zero added to 19.2?

3. Write the problem $40.3 - 6.54$ in the grid.

4. Did you need to add a zero as a placeholder? If so, where?

5. Subtract 6.54 from 40.3 using the grid. What is the answer? _____

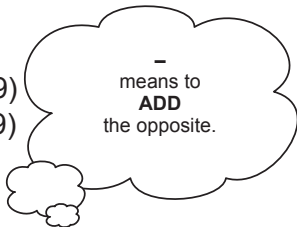
LESSON
3-3

Subtracting Rational Numbers

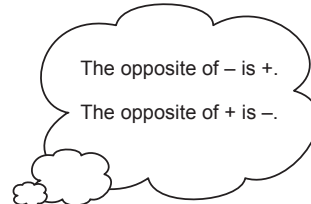
Success for English Learners

Problem 1

$$\begin{aligned} 5 - 9 &= 5 - (+9) \\ &= 5 + (-9) \\ &= -4 \end{aligned}$$

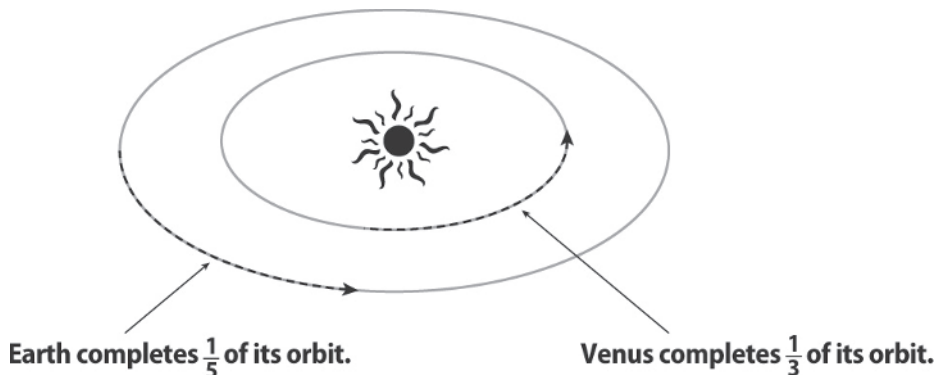


$$\begin{aligned} -4 - 3 &= -4 - (+3) \\ &= -4 + (-3) \\ &= -7 \end{aligned}$$



Problem 2

From January 1 to March 14



$\frac{1}{3} - \frac{1}{5}$ describes how much more of its orbit Venus completes than Earth does.

1. Look at Problem 1. What is the opposite of 9? _____
2. In the second part of Problem 1, why don't you change the -4 to $+4$?

3. Is $3 - 5$ the same as $5 - 3$? Explain.

4. In Problem 2, what is the first thing you need to do to find $\frac{1}{3} - \frac{1}{5}$?

5. How much more of its orbit does Venus complete? _____

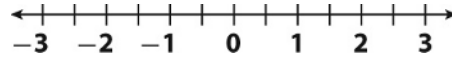
LESSON
3-4

Multiplying Rational Numbers

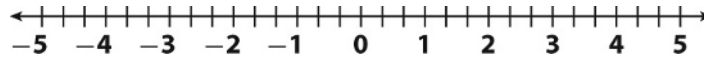
Practice and Problem Solving: A/B

Use the number line to find each product.

1. $4\left(-\frac{1}{2}\right)$ _____



2. $-5\left(-\frac{2}{3}\right)$ _____



Find the product.

3. $-2(3.1)$

4. $4(-5.4)$

5. $-3.3(6)$

6. $-3(-5.6)$

7. $4.5(8)$

8. $2(-1.05)$

9. $-2.05(4)$

10. $-3.5(-9)$

Find the product. Show your work.

11. $\left(\frac{2}{3}\right) \times (-6) \times 5 =$ _____

12. $\left(-\frac{3}{5}\right) \left(-\frac{10}{3}\right) \left(-\frac{2}{9}\right) =$ _____

13. $-7 \times \left(-\frac{3}{5}\right) \times \left(\frac{15}{7}\right) =$ _____

14. $2(4)\left(\frac{1}{16}\right) =$ _____

Solve. Show your work.

15. A landscaper installs 12 sections of trellis. Each section of trellis is $\frac{3}{4}$ yard long. How many yards of trellis are installed altogether?

16. A biologist uses a box-shaped fish trap that measures $\frac{1}{4}$ -meter by $\frac{2}{3}$ -meter by $\frac{3}{5}$ -meter. What is the volume of the trap in cubic meters?

17. The temperature at noon is 75°F . The temperature drops 3 degrees every half hour. What is the temperature at 4 P.M.?

LESSON
3-4**Multiplying Rational Numbers****Practice and Problem Solving: C**

Compare the products by writing $<$ or $>$. Without doing the calculations, explain how you know your answers are correct.

1. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^3$ 1

2. $\left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3$ 0

3. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ $\left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)$

4. $0.5(-1.1)$ 0

Tell whether each statement is *True* or *False*. Without doing the calculations, explain how you know your answers are correct.

5. $\left(-\frac{1}{2}\right)^6 < 0$ _____

6. $(1.5)^7 > 1$ _____

7. $(0.9)^4 > 1$ _____

Solve. (The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.)

8. The radius of a spherical balloon is $\frac{1}{2}$ foot. The radius of a second one is $\frac{3}{4}$ foot. How do the volumes of the balloons compare?

9. The radius of a sphere is reduced by one third. How does its volume change?

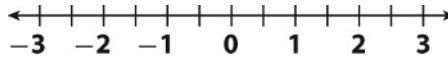
LESSON
3-4

Multiplying Rational Numbers

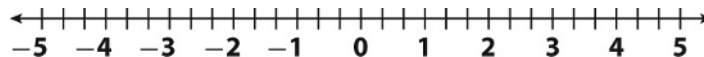
Practice and Problem Solving: D

Fill in the blanks to complete the computation. Refer to the number line if you need help. The first one in each group is done for you.

1. $6\left(-\frac{1}{2}\right) = \frac{\left(-\frac{1}{2}\right)}{\underline{\hspace{1cm}}} + \frac{\left(-\frac{1}{2}\right)}{\underline{\hspace{1cm}}} + \frac{\left(-\frac{1}{2}\right)}{\underline{\hspace{1cm}}} + \frac{\left(-\frac{1}{2}\right)}{\underline{\hspace{1cm}}} + \frac{\left(-\frac{1}{2}\right)}{\underline{\hspace{1cm}}} + \frac{\left(-\frac{1}{2}\right)}{\underline{\hspace{1cm}}} = \underline{-\frac{6}{2}} \text{ or } \underline{-3}$



2. $-3\left(-\frac{2}{3}\right) = -\underline{\hspace{1cm}} - \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$



Write each addition expression as a multiplication expression. Then, solve. The first one is done for you.

3. $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} = \underline{4} \times \frac{\underline{5}}{\underline{8}} = \underline{\frac{20}{8}} \text{ or } \underline{2\frac{1}{2}}$

4. $(-2.5) + (-2.5) = \underline{\hspace{2cm}} = \underline{\hspace{1cm}}$

5. $\left(-\frac{2}{9}\right) + \left(-\frac{2}{9}\right) + \left(-\frac{2}{9}\right) = \underline{\hspace{2cm}} = \underline{\hspace{1cm}}$

Fill in the missing steps. Then, find the answer. The first one is started for you.

6. $\left(-\frac{1}{4}\right) \times \frac{3}{5} \times \left(-\frac{2}{5}\right) = \underline{-\frac{1}{4}} \times \frac{\left(\frac{3 \times (-2)}{5 \times 5}\right)}{\underline{\hspace{1cm}}} = \underline{-\frac{1}{4}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

7. $4(2.5)0.8 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times 0.8 = \underline{\hspace{1cm}} \times 0.8 = \underline{\hspace{1cm}}$

Solve. The first one is started for you.

8. A seal dives when it see a whale. The seal dives for 5 seconds at an average rate of 3.5 meters per second.

a. Write an addition expression to represent how far the seal dives in 5 seconds. Find the sum.

$\underline{(-3.5) + (-3.5) + (-3.5) + (-3.5) + (-3.5)} = \underline{\hspace{2cm}}$

b. Write a multiplication expression to represent how far the seal dives in 5 seconds. Find the product.

LESSON
3-4**Multiplying Rational Numbers****Reteach**

You can use a number line to multiply rational numbers.

$$5 \times \left(-\frac{1}{2}\right)$$

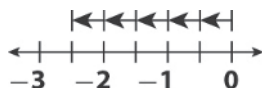
How many times is the $-\frac{1}{2}$ multiplied?

Five times, so there will be 5 jumps of $\frac{1}{2}$ unit each along the number line.

Your first jump begins at 0. In which direction should you move?

$-\frac{1}{2}$ is negative, and 5 is positive. They have different signs. So, each jump will be to the *left*.

(When both numbers have the same sign, each jump will be to the *right*.)



Name the numbers where each jump ends, from the first to the fifth jump.

$$-\frac{1}{2}, -1, -1\frac{1}{2}, -2, -2\frac{1}{2}$$

$$\text{So, } 5 \times \left(-\frac{1}{2}\right) = -2\frac{1}{2}.$$

Find each product. Draw a number line for help.

1. $6 \times \frac{1}{4}$

Multiply $\frac{1}{4}$ how many times? _____

Which direction on the number line? _____

Move from 0 to where? _____ Product: _____

2. $-8(-3.3)$

Multiply (-3.3) how many times? _____

Move from 0 to where? _____ Product: _____

3. 4.6×5

Multiply 4.6 how many times? _____

Move from 0 to where? _____ Product: _____

LESSON
3-4

Multiplying Rational Numbers

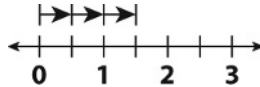
Reading Strategies: Understand Symbols

Each number and symbol in a multiplication problem tells you something about how to use a number line to solve it.

$-3\left(-\frac{1}{2}\right)$ ← The $\frac{1}{2}$ *inside* the parentheses signals that each jump along a number line will be half a unit.

$-3\left(-\frac{1}{2}\right)$ ← The 3 *outside* the parentheses signals that there will be a total of 3 jumps.

$-3\left(-\frac{1}{2}\right)$ ← Both numbers have the same sign in front of them, a “-” sign. So, in this case, each jump along the number line will be to the *right*, starting from 0.



$$\text{So, } -3\left(-\frac{1}{2}\right) = 1\frac{1}{2}.$$

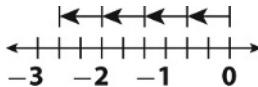
Put the steps together.

$$4\left(-\frac{2}{3}\right)$$

Step 1: The number inside the parentheses is the length of each jump along the number line. Here, each jump is $\frac{2}{3}$.

Step 2: The 4 outside the parentheses signals that a total of 4 jumps must be made.

Step 3: The two numbers have different signs, so each jump along the number line is to the left.



$$\text{So, } 4\left(-\frac{2}{3}\right) = -\frac{8}{3} \text{ or } -2\frac{2}{3}.$$

Use the steps to read the numbers and symbols. Then, multiply.

1. $\left(-\frac{2}{3}\right)(-6)$

2. $-7(0.75)$

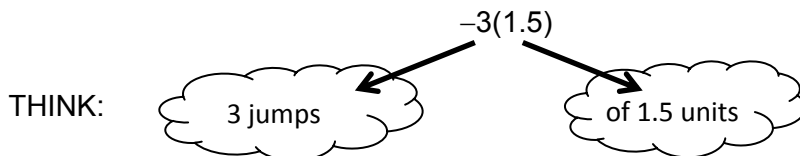
3. $4\left(\frac{1}{5}\right)(-3)$

LESSON
3-4 **Multiplying Rational Numbers**
Success for English Learners

Problem 1

Multiply: $-3(1.5)$

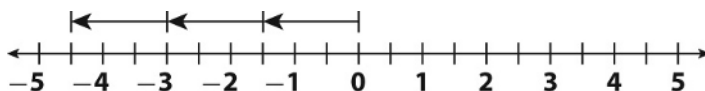
Use a number line.



Start at 0. Jump which way?

Rule 1: A negative (-) times a positive (+) makes a negative (-).

The product will be negative, so jump *left*.



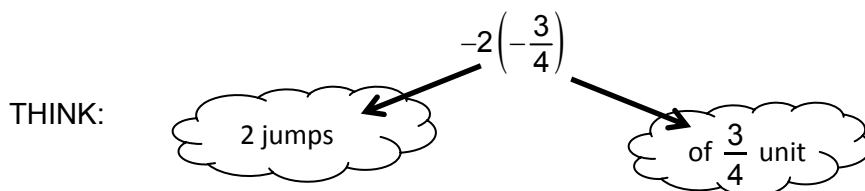
So, $-3(1.5) = -4.5$

Use Rule 1 to multiply.

1. $-2.2(4)$ _____ 2. $-6(0.5)$ _____ 3. $-1(9.9)$ _____ 4. $-3.3(3)$ _____

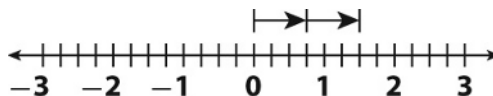
Problem 2

Multiply: $-2\left(-\frac{3}{4}\right)$



Rule 2: A negative (-) times a negative (-) makes a positive (+).

The product will be positive, so jump *right*.



So, $-2\left(-\frac{3}{4}\right) = 1\frac{2}{4} = 1\frac{1}{2}$.

Use Rule 2 to multiply.

5. $-\frac{6}{5}(-10) =$ _____ 6. $-4\left(-\frac{3}{7}\right) =$ _____ 7. $\left(-\frac{1}{4}\right)(-16) =$ _____

LESSON
3-5**Dividing Rational Numbers****Practice and Problem Solving: A/B****Find each quotient.**

1. $\frac{1}{2} \div (-3)$

2. $-6 \div \left(-\frac{3}{4}\right)$

3. $\frac{5}{6} \div 10$

4. $\frac{5.25}{15}$

5. $24 \div (-3.2)$

6. $-0.125 \div (-0.5)$

7. $-\frac{1}{7} \div -\frac{3}{14}$

8. $\frac{\left(\frac{3}{2}\right)}{\left(-\frac{9}{8}\right)} =$

9. $-1\frac{1}{2} \div 3\frac{1}{3}$

10. $2\frac{1}{4} \div \frac{3}{8}$

11. $\frac{4.2}{-2.4}$

12. $-\frac{5}{8} \div \left(-\frac{5}{16}\right)$

Fill in the blank with a number to make a true statement.

13. $0.25 \div \underline{\hspace{2cm}} = -0.25$

14. $-\frac{1}{2} \div \underline{\hspace{2cm}} = -\frac{7}{3}$

15. $\frac{1}{7} \div \underline{\hspace{2cm}} = 14$

Write a division problem for each situation. Then, solve it.

16. How many quarter-pound
- $\left(\frac{1}{4}\right)$
- packets of plant food can a garden shop make out of 8 pounds of the plant food?
-
- _____

17. The assembly of a machine takes
- $\frac{3}{4}$
- hour. There are twelve steps in the assembly process. What is the average time for each step?
-
- _____

18. A 35-meter length of cable is cut into pieces that measure 1.25 meters each. Into how many pieces is the cable cut?
-
- _____

- 19.
- $4\frac{1}{8}$
- tons of gravel is spread evenly across
- $2\frac{1}{6}$
- acres. How many tons of gravel are on each acre?
-
- _____

LESSON
3-5

Dividing Rational Numbers

Practice and Problem Solving: C

Write each quotient two other ways by moving the negative sign.

1. $\frac{-1}{5} =$ _____ 2. $-\frac{7}{30} =$ _____ 3. $\frac{1}{-2} =$ _____
 = _____ = _____ = _____

Use $>$, $<$, or $=$ to compare the quotients. Show the quotients.

4. $(4 \times \frac{1}{3}) \div \frac{2}{5} \bigcirc 4 \times (\frac{1}{3} \div \frac{2}{5})$ 5. $(4.5 \div 0.5) \div 3 \bigcirc 4.5 \div (0.5 \div 3)$

6. $(6 \div -\frac{1}{5}) \times -\frac{4}{3} \bigcirc 6 \div (-\frac{1}{5} \times -\frac{4}{3})$ 7. $5.5(-3 \times 7.5) \bigcirc 7.5(-3 \times 5.5)$

Use decimals to rewrite each quotient. Then, solve.

8. $\frac{(\frac{2}{5})}{(-\frac{5}{8})}$ 9. $\frac{(-5\frac{2}{5})}{(-\frac{5}{16})}$ 10. $\frac{(\frac{1}{4})}{(\frac{3}{5})}$

Write each quotient as a decimal and as a fraction. Show your work.

11. $\frac{(1 + \frac{1}{2} + \frac{1}{4})}{(1 - \frac{1}{2} - \frac{1}{4})}$ _____

12. $\frac{(1 + \frac{1}{3} + \frac{1}{6})}{(1 - \frac{1}{3} - \frac{1}{6})}$ _____

13. $\frac{(1 + \frac{1}{4} + \frac{1}{8})}{(1 - \frac{1}{4} - \frac{1}{8})}$ _____

14. If the 4s in question 13 are replaced with 5s, and the 8s are replaced with 10s, how will the quotient compare to the other three quotients? Explain.

LESSON
3-5

Dividing Rational Numbers

Practice and Problem Solving: D

Write the missing number. Then, solve. The first one is done for you.

$$1. -6 \div \frac{3}{4} = -6 \times \frac{4}{\underline{3}}$$

$$\underline{-8}$$

$$2. \frac{4}{5} \div 8 = \frac{4}{5} \times \underline{\hspace{2cm}}$$

$$3. \frac{-7}{8} \div \frac{7}{-4} = \frac{-7}{8} \times \underline{\hspace{2cm}}$$

$$4. \frac{\left(\frac{5}{-3}\right)}{\left(\frac{7}{8}\right)} = \left(\frac{5}{-3}\right) \times \underline{\hspace{2cm}}$$

$$5. \frac{-2}{\left(\frac{4}{9}\right)} = -2 \times \underline{\hspace{2cm}}$$

$$6. \frac{\left(-4\frac{3}{4}\right)}{4} = -\frac{19}{4} \times \underline{\hspace{2cm}}$$

Solve. The first one is done for you.

$$7. \frac{-1}{8} \div -5 = \frac{-1}{8} \times -\frac{1}{5} = \underline{\hspace{2cm}}$$

$$\frac{1}{40}$$

$$8. \frac{\left(2\frac{1}{4}\right)}{\left(-\frac{6}{7}\right)} = \frac{9}{4} \times -\frac{7}{6} = \underline{\hspace{2cm}}$$

$$9. 3 \div \frac{6}{7} = \frac{3}{1} \times \frac{7}{6} = \underline{\hspace{2cm}}$$

Find the missing numbers. Add zeros as needed. Then, solve.
The first one is done for you.

$$10. 0.4 \div 2.5 \rightarrow 2.5 \overline{) \hspace{1cm}}?$$

$$\underline{0.40; 0.16}$$

$$11. \frac{-4.75}{0.3} \rightarrow ? \overline{) -4.75}$$

$$12. 8 \div 2.5 \rightarrow 2.5 \overline{) \hspace{1cm}}?$$

Solve.

13. The town's highway department marks a new road with reflective markers. The road is $6\frac{3}{4}$ miles in length. The markers are spaced every eighth $\left(\frac{1}{8}\right)$ of a mile.

a. Write a division problem to find how many markers are installed.

b. Solve the problem.

c. Explain the answer.

LESSON

3-5

Dividing Rational Numbers**Reteach**

To divide fractions:

- Multiply the first, or “top,” number by the reciprocal of the second, or “bottom,” number.
- Check the sign.

Divide: $-\frac{3}{5} \div \frac{2}{3}$

Step 1: Rewrite the problem to multiply by the reciprocal.

$$-\frac{3}{5} \div \frac{2}{3} = -\frac{3}{5} \times \frac{3}{2}$$

Step 2: Multiply.

$$-\frac{3}{5} \times \frac{3}{2} = \frac{-3 \times 3}{5 \times 2} = \frac{-9}{10}$$

Step 3: Check the sign.

A negative divided by a positive is a negative.

So, $\frac{-9}{10}$ is correct.

$$-\frac{3}{5} \div \frac{2}{3} = -\frac{9}{10}$$

Write the sign of each quotient.

1. $4\frac{1}{4} \div 3\frac{1}{2}$

2. $-3.5 \div 0.675$

3. $\frac{5}{\left(-\frac{3}{5}\right)}$

4. $-\frac{2}{9} \div \left(-\frac{3}{8}\right)$

Complete the steps described above to find each quotient.

5. $-\frac{1}{7} \div \left(-\frac{5}{9}\right)$

6. $\frac{7}{8} \div \frac{8}{9}$

Step 1: _____

Step 1: _____

Step 2: _____

Step 2: _____

Step 3: _____

Step 3: _____

LESSON
3-5

Dividing Rational Numbers

Reading Strategies: Read a Table

Writing the signs of the numbers in a table can help you write the correct sign for the answer to division problems.

Look at these rules for signs for two numbers, A and B :

If A is	and B is	then $A \div B$ is
+	+	+
+	-	-
-	-	+

Write the correct sign for each quotient: + or -.

	-4	8.25	$-\frac{6}{11}$	$2\frac{1}{12}$
$-7.5 \div$	1. _____	2. _____	3. _____	4. _____
$\frac{3}{8} \div$	5. _____	6. _____	7. _____	8. _____
$-5\frac{2}{3} \div$	9. _____	10. _____	11. _____	12. _____

The rules for signs apply for more than two numbers. Write + or - in the empty cells for the division problem $(A \div B) \div C$.

If A	B	and C	then $(A \div B) \div C$ is
+	-	13. _____ ,	+
+	-	+	14. _____.
-	-	15. _____ ,	+

LESSON
3-5

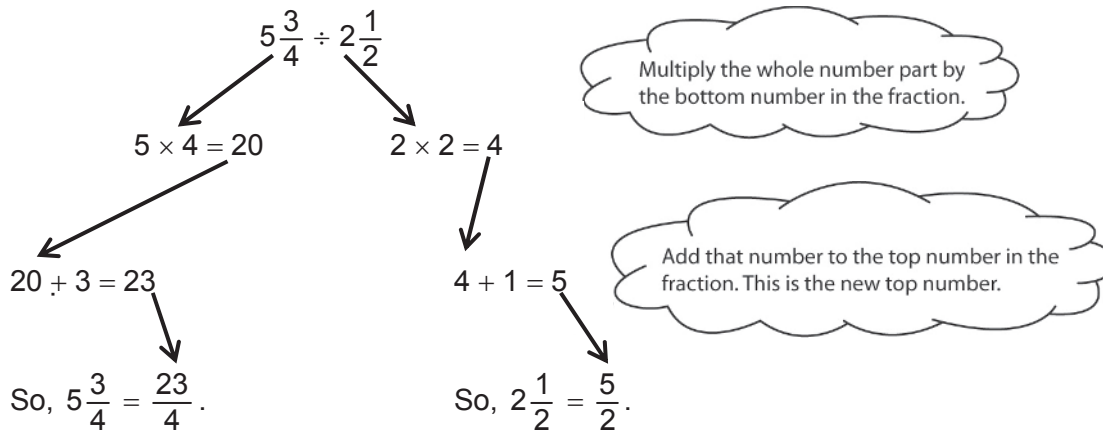
Dividing Rational Numbers

Success for English Learners

Problem 1

How to divide mixed numbers . . .

Step 1: Rewrite each mixed number as an improper fraction.



Step 2: Which fraction are you dividing by? “Flip” it to get the reciprocal.

“Flip” $\frac{5}{2}$ \longrightarrow $\frac{2}{5}$

Step 3: Multiply the first fraction by this “flipped” fraction:

$$\frac{23}{4} \times \frac{2}{5} = \frac{23 \times 2}{4 \times 5} = \frac{46}{20}$$

What number can 46 and 20 both be divided by?

Step 4: Simplify the fraction.

$$\frac{46}{20} = \frac{46 \div 2}{20 \div 2} = \frac{23}{10}$$

Step 5: Change that improper fraction back to a mixed number.

$$\frac{23}{10} = 2\frac{3}{10}$$

How many 10s in 23?
How many left over?

Solve. Use the 5 steps.

1. $7\frac{5}{8} \div 3\frac{2}{3}$ _____

2. $6\frac{1}{5} \div 3\frac{1}{10}$ _____

LESSON
3-6**Applying Rational Number Operations****Practice and Problem Solving: A/B**

Estimate each answer. Explain your reasoning.

1. Sections of prefabricated fencing are each $4\frac{1}{3}$ feet long. How long are $6\frac{1}{2}$ sections placed end to end?

2. One half liter of lemonade concentrate is added to 3 liters of water. How many $\frac{1}{3}$ -liter servings of lemonade are made?

3. Two $2\frac{1}{2}$ -inch plastic strips and two $5\frac{1}{3}$ -inch plastic strips are used to form a rectangle. What is the perimeter of the rectangle?

4. The average mass of the eggs laid by chickens on Ms. Watson's farm is 3.5 grams. About how many grams does a dozen eggs weigh?

5. An 8.5-centimeter green bean pod contains peas that average 0.45-centimeter in diameter. How many peas are in the pod?

Solve by converting to the easiest form of the rational numbers to use in the problem. Show your work

6. Arwen uses a dropper that produces drops that have a volume of $\frac{1}{8}$ -milliliter to fill a 30-milliliter test tube. How many drops does it take to fill the test tube?

7. Three strips of 2-yard-wide outdoor carpet are used to cover a sidewalk. One is 3.5 yards long, the second is 25 percent longer than the first, and the third is $6\frac{1}{4}$ yards long. How long are the three carpets placed end to end?

LESSON
3-6**Applying Rational Number Operations*****Practice and Problem Solving: C***

Use the information given in the table below to complete Exercises 1–4.

Planets' Orbital Velocity

Planet	Orbital Velocity (mi/s)
Mercury	29.74
Venus	21.76
Earth	18.5
Mars	14.99
Jupiter	8.12
Saturn	6.02
Uranus	4.23

Solve. Show your work. Express the answer in fraction form.

1. How many miles does Mercury travel in an hour? Simplify the fractions in your answer.

2. How much greater is the orbital velocity of Mercury than Jupiter? Simplify the fractions in your answer.

3. During the time it takes Saturn to travel 32,508 miles, how much time in seconds has elapsed on Earth? Simplify the fractions in your answer.

4. How many miles does Venus travel in a minute? Simplify the fractions in your answer.

LESSON
3-6

Applying Rational Number Operations

Practice and Problem Solving: D

Solve. Show your work. The first one has been done for you.

1. A middle school conducts a recycling drive, during which $\frac{1}{5}$ of the materials collected were bottles and $\frac{1}{4}$ was paper. Cardboard boxes made up $\frac{1}{10}$ of the material. How much of the total do these three categories of items represent?

The LCM of the three denominators, 4, 5, and 10, is 20.

Multiply each fraction to get a common denominator of 20.

$$\frac{1}{5} \cdot \frac{4}{4} = \frac{4}{20} \qquad \frac{1}{4} \cdot \frac{5}{5} = \frac{5}{20} \qquad \frac{1}{10} \cdot \frac{2}{2} = \frac{2}{20}$$

Add the fractions: $\frac{4}{20} + \frac{5}{20} + \frac{2}{20} = \frac{11}{20}$

Bottles, paper, and cardboard boxes were $\frac{11}{20}$ of the total amount of recycled material collected by the middle school.

2. A family budgets $\frac{1}{2}$ of its income for housing and $\frac{1}{3}$ for food. What fraction of their budget do these expenses cover?

3. Decorations for the seventh-grade dance take $\frac{1}{6}$ of the student council's budget. Entertainment takes $\frac{3}{8}$ of the budget. What fraction of the budget do these expenses cover? What fraction is left for other activities? (*Hint:* To answer the second question, subtract the answer to the first question from 1, which represents the whole budget.)

LESSON
3-6

Applying Rational Number Operations

Reteach

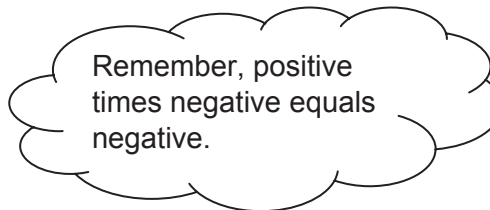
To multiply fractions and mixed numbers:

Step 1: Write any mixed numbers as improper fractions.

Step 2: Multiply the numerators.

Step 3: Multiply the denominators.

Step 4: Write the answer in simplest form.

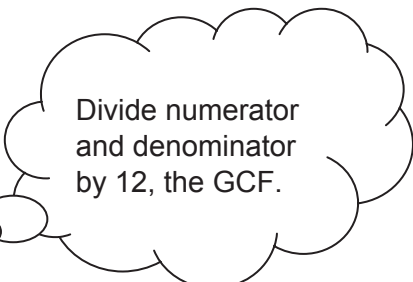


Multiply: $\frac{4}{9} \cdot \frac{8}{3}$

$$\frac{4}{9} \cdot \frac{3}{8} = \frac{4 \cdot 3}{9 \cdot 8}$$

$$= \frac{12}{72}$$

$$= \frac{1}{6}$$



Multiply: $6\frac{1}{4} \cdot \left(-1\frac{4}{5}\right)$

$$6\frac{1}{4} \cdot \left(-1\frac{4}{5}\right) = \frac{25}{4} \cdot \left(\frac{-9}{5}\right)$$

$$= \frac{25 \cdot (-9)}{4 \cdot 5}$$

$$= \frac{-225}{20}$$

$$= -11\frac{1}{4}$$

Use the models to solve the problems.

1. One cup of dog food weighs $1\frac{4}{5}$ ounces. A police dog eats $6\frac{1}{3}$ cups of food a day. How many ounces of food does the dog eat each day?

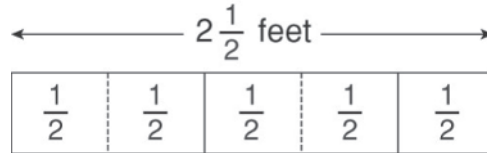
2. A painter spends 3 hours working on a painting. A sculptor spends $2\frac{2}{3}$ as long working on a sculpture. How long does the sculptor work?

3. A meteorite found in the United States weighs $\frac{7}{10}$ as much as one found in Mongolia. The meteorite found in Mongolia weighs 22 tons. How much does the one found in the United States weigh?

4. A chicken salad recipe calls for $\frac{1}{8}$ pound of chicken per serving. How many pounds of chicken are needed to make $8\frac{1}{2}$ servings?

LESSON
3-6**Applying Rational Number Operations****Reading Strategies: Use a Visual Model**

The Smith family has a two-and-a-half-foot-long sandwich to share. One-half foot of the sandwich will serve one person. How many one-half foot servings are in this sandwich?



Use the model to answer each question.

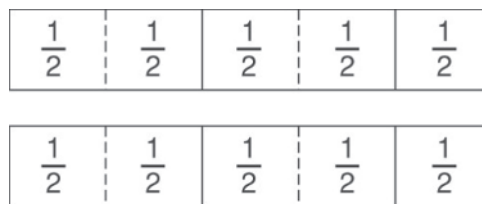
- How long is the sandwich?

- How long is each serving?

- If you divided the sandwich into $\frac{1}{2}$ -foot servings, how many would you have?

- What is $2\frac{1}{2} \div \frac{1}{2}$?

Suppose you have two sandwiches.



- How many feet are in both sandwiches?

- What is $2\frac{1}{2} \times 2$?

- Compare the answers to $2\frac{1}{2} \div \frac{1}{2}$ and $2\frac{1}{2} \times 2$. What do you notice?

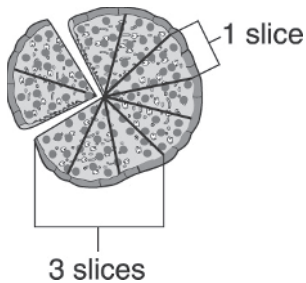
LESSON
3-6

Applying Rational Number Operations

Success for English Learners

Problem 1

Add $\frac{3}{10} + \frac{1}{10}$.



$\frac{10}{10} = 1$ whole pizza

$3 + 1 = 4$ slices

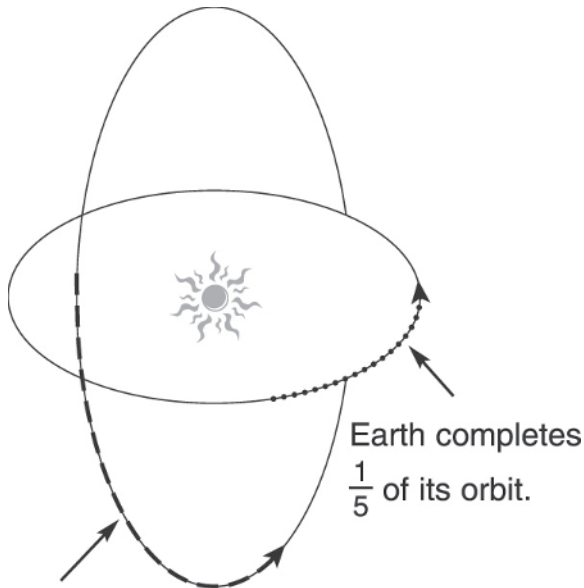
$\frac{3}{10}$ of the pizza + $\frac{1}{10}$ of the pizza

$\frac{3}{10} + \frac{1}{10} = \frac{4}{10}$ of the pizza

Simplify. $\frac{4}{10} = \frac{2}{5}$.

Problem 2

From January 1 to March 14:



Venus completes $\frac{1}{3}$ of its orbit.

$\frac{1}{3} - \frac{1}{5}$ describes how much more of its orbit Venus completes than Earth.

1. Explain what the denominator represents in Problem 1.

2. In order to find the answer to $\frac{1}{3} - \frac{1}{5}$ in Problem 2, what is the first thing you need to do?

3. If the denominators of two fractions are the same, how do you add the fractions?

MODULE
3
Rational Numbers
Challenge

1. A meteorologist is measuring the change in temperature in three cities. For five days, she recorded the daily temperature in each city.

Temperatures (°C) in Three Cities

City	Monday	Tuesday	Wednesday	Thursday	Friday
City A	$-\frac{1}{8}$	$2\frac{1}{4}$	$-3\frac{1}{2}$	$5\frac{4}{5}$	$-12\frac{1}{2}$
City B	$4\frac{1}{5}$	$-1\frac{3}{5}$	$-8\frac{1}{10}$	$11\frac{1}{5}$	$3\frac{3}{10}$
City C	$11\frac{1}{3}$	$2\frac{5}{6}$	$-3\frac{2}{3}$	$-9\frac{1}{6}$	$2\frac{1}{3}$

The temperature change is the difference between the temperature on a certain day and the temperature the day before.

For example, in City A the temperature change from Monday to

Tuesday was $2\frac{1}{4} - \left(-\frac{1}{8}\right) = 2\frac{3}{8}$.

The meteorologist defines the “temperature volatility” to be the sum of the daily temperature changes over a certain period of time.

Which city had the greatest temperature volatility for the five-day period shown?

2. In the mathematical expression below, the three boxes can be filled with any operation symbol (+, −, ×, or ÷).

$$2 \square -\frac{1}{8} \square -10 \square 16$$

Each operation symbol can only be used once.

For example, the boxes can be filled as shown below:

$$2 \div -\frac{1}{8} \square -10 \div 16, \text{ which equals } 10.$$

How can the operations be placed in the boxes to yield the greatest possible value for the expression? (Hint: Remember to multiply and divide before you add and subtract.)

UNIT 1: The Number System

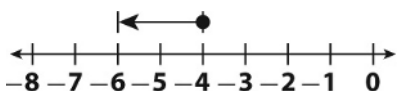
MODULE 1 Adding and Subtracting Integers

LESSON 1-1

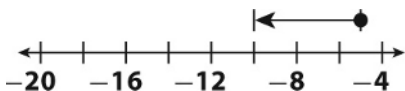
Practice and Problem Solving: A/B

- a. 8
b. negative
c. -8
- a. 11
b. negative
c. -11

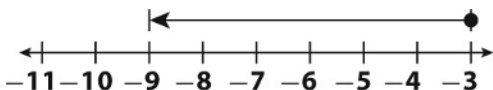
3. -6



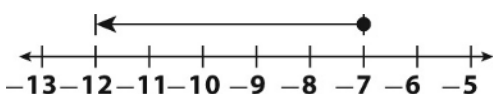
4. -10



5. -9



6. -12



- 8
- 9
- 53
- 93
- 224
- 95
- 600
- 1310
- $-3 + (-2) + (-4) = -9$; -9 feet. The hole is 9 feet deep.

Practice and Problem Solving: C

- a. $-42 + (-87) + (-29) = -158$
b. $-57 + (-75) + (-38) = -170$
c. The store had more red apples left over. The store started with the same number

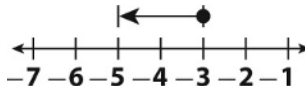
of red apples and green apples. It sold more green apples than red apples, so it had more red apples left.

- a. $-2 + (-3) + (-13) = -18$
b. The hotel guest got off on the 14th floor. The manager started on the 19th floor and rode 2 floors down to the 17th floor when the hotel guest got on. They rode the elevator down 3 floors. $17 - 3 = 14$, so the hotel guest got off on the 14th floor.

Practice and Problem Solving: D

- a. 7
b. positive
c. +7
- a. 10
b. negative
c. -10

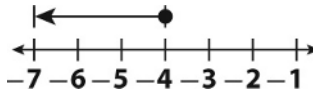
3. -5



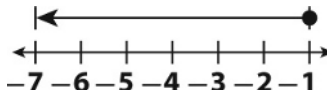
4. -6



5. -7



6. -7



- 4
- 8
- 19
- 35
- \$8

Reteach

- a. positive
b. $3 + 6 = 9$
c. 9

2. a. negative
 - b. $7 + 1 = 8$
 - c. -8
3. a. negative
 - b. $5 + 2 = 7$
 - c. -7
4. a. positive
 - b. $6 + 4 = 10$
 - c. 10
5. -13
6. -16
7. 37
8. -41
9. -24
10. 52

Reading Strategies

1. Each counter represents -1 .
2. Each counter represents a dollar that Sarah withdrew. The counters make it is easier to see how many dollars Sarah withdrew each day.
3. You can simply count the counters to find the sum.
4. $-3 + (-5) + (-4) + (-1) = -13$

Success for English Learners

1. positive counters
2. because you are adding a negative number
3. Answers will vary. Sample answer: Erica bought stamps three times this week. She bought 5 stamps on Monday, 3 stamps on Wednesday, and 4 stamps on Friday. How many stamps did Erica buy this week? ($5 + 3 + 4 + 12$)

LESSON 1-2

Practice and Problem Solving: A/B

1. -1
2. 1
3. 5
4. -1
5. -1
6. -3

7. -2
8. 4
9. 8
10. 2
11. 43
12. 21
13. -29
14. -10
15. 11°F
16. 3 yards
17. -9 points
18. a. negative
 - b. loss of 6, or -6

Practice and Problem Solving: C

1. negative; -10
2. positive; 5
3. negative; -7
4. positive; 5
5. positive; 6
6. positive; 15
7. negative; -1
8. positive; 1
9. the same sign as the integers
10. It is the sign of the integer whose absolute value is greater.
11. -15
12. -24
13. 13
14. -30
15. 0
16. -18
17. -5°F
18. \$150
19. Rita; 11 points

Practice and Problem Solving: D

1. -1
2. -7
3. -5
4. -1
5. -1

6. 12
7. 4
8. 8
9. -5
10. -10
11. -6
12. 5°F
13. -22°F
14. -97 ft
15. 17,500 ft

Reteach

1. subtract; the numbers have different signs
2. negative
3. 4
4. -5
5. -1
6. -4
7. 2
8. -5
9. 9
10. -10
11. -16
12. Sample answer: I look at 3 and 9 and see that $9 > 3$. Since the sign on 9 is negative, the answer is negative.

Reading Strategies

1. 0
2. right; 6
3. left; 4
4. 2
5. 0
6. left; 5
7. left; 3
8. -8

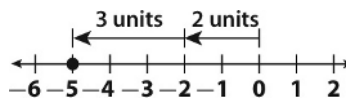
Success for English Learners

1. negative number
2. No, the sum can be positive or negative.
3. negative
4. positive

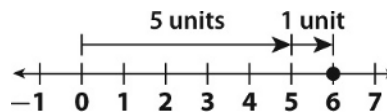
LESSON 1-3

Practice and Problem Solving: A/B

1. -5



2. 6



3. -10
4. 5
5. -4
6. 24
7. 0
8. 46
9. -1
10. 42
11. -6
12. -26
13. 30
14. -5
15. 9°C
16. 14°F
17. 4°C
18. 7°C
19. 240°C

Practice and Problem Solving: C

1. 16
2. -22
3. 7
4. 0
5. 29
6. 9
7. -2
8. 0
9. -10
10. when $x < y$
11. when $x > y$
12. 12°F, -2°F
13. Pacific; 2,400 m

14. 11,560; -185 ; -185 is closer to sea level; 11,375 ft
15. Saturday
16. 1°

Practice and Problem Solving: D

1. -5
2. -4
3. -7
4. -5
5. 6
6. -16
7. 0
8. 1
9. 7
10. 16
11. -11
12. 610°C
13. \$35,000
14. 9°F

Reteach

1. a. 5
b. -1
c. 20
2. a. negative
b. 2
c. -2
3. 40
4. -3
5. -26
6. 0
7. 31
8. -5

Reading Strategies

1. left
2. 7
3. right
4. 3
5. -4
6. right; 2
7. left; 6
8. -4

Success for English Learners

1. positive
2. negative

LESSON 1-4

Practice and Problem Solving: A/B

1. $-2 - 19 + 7 = -14$; 14 feet below the surface of the water
2. $45 - 8 + 53 - 6 = 84$; 84 points
3. 20
4. -27
5. 18
6. 110
7. 52
8. 34
9. $<$
10. $>$
11. a. $225 + 75 - 30 = 270$; 270 points
b. Maya

Practice and Problem Solving: C

1. $-35 - 29 + 7 - 10 = -67$; Jana is 67 ft from the end of the fishing line.
2. a. $500 + 225 - 105 + 445 = 1065$; 1065 ft above the ground
b. Kirsten is closer to the ground; Gigi's balloon position is $500 + 240 - 120 + 460 = 1080$ ft, which is greater than 1065 ft.
3. a. $20 + 20 + 30 + 30 - 10 - 10 - 10 = 100$; 100 points
b. David and Jon tied. Jon scored $20 + 20 + 20 + 30 + 30 - 10 - 10 = 100$, or 100 points, which is the same number of points that David scored.

Practice and Problem Solving: D

1. $-2 - 9 + 3 = -8$; 8 ft below the surface of the water
2. $20 - 5 + 10 = 25$; 25 points
3. -1
4. -24
5. 20
6. -9

7. 8
 8. 100
 9. <
 10. >
 11. $200 - 30 + 70 = 240$; 240 points

Reteach

1. a. $10 + 5 - 19$
 b. $15 - 19 = -4$
 c. -4
 2. a. $14 - 15 - 3$
 b. $14 - 18 = -4$
 c. -4
 3. a. $10 - 80 - 6$
 b. $10 - 86 = -76$
 c. -76
 4. a. $7 + 13 - 21$
 b. $20 - 21 = -1$
 c. -1
 5. a. $13 + 2 - 5 - 6$
 b. $15 - 11 = 4$
 c. 4

6. a. $18 + 6 - 4 - 30$
 b. $24 - 34 = -10$
 c. -10

Reading Strategies

1. +700; above
 2. when the balloon rises; rise
 3. when the balloon drops; drop
 4. $700 - 200 + 500 - 100 = 900$
 5. 900 ft above the ground
 6. Angelo is higher than where he started because 900 is greater than 700.

Success for English Learners

1. When money is withdrawn, it is taken out of the bank account. So, you subtract.
 2. When money is deposited, it is put into the bank account. So, you add.
 3. Answers may vary. Sample answer: Jose has \$25. He spends \$5, and then earns \$15. How much money does Jose have at the end? ($25 - 5 + 15 = 35$; \$35)

MODULE 1 Challenge

1. Calculate the difficulty using the method shown in the example.

Trail	Mile 1	Mile 2	Mile 3	Mile 4	Mile 5	Total
Breakneck	$100 - (-2) = 102$	$-2 - 100 = -102$	$150 - (-2) = 152$	$-8 - 150 = -158$	$250 - (-8) = 258$	252
Lake Shore	$0 - (-10) = 10$	$6 - 0 = 6$	$55 - 6 = 49$	$-1 - 55 = -56$	$60 - (-1) = 61$	70
Mountain View	$-2 - 40 = -42$	$120 - (-2) = 122$	$35 - 120 = -85$	$200 - 35 = 165$	$180 - 200 = -20$	140

The most difficult trail is Breakneck.

2. The greatest possible value is obtained by filling the boxes as follows.

$-3 \boxed{+} 5 \boxed{-} -4 \boxed{-} -10 \boxed{+} 18 = 34$

MODULE 2 Multiplying and Dividing Integers

LESSON 2-1

Practice and Problem Solving: A/B

1. -80
2. -72
3. 40
4. -39
5. 0
6. -80
7. 189
8. -11
9. -72
10. 80
11. -54
12. 49
13. $4(-6) = -24$; -24 points
14. $5(-3) = -15$; -15°
15. $8(-18) = -144$; $200 + (-144) = 56$; \$56
16. $3(-5) = -15$; $8 + (-15) = -7$; -7°
17. $6(-25) = -150$; $325 + (-150) = 175$; \$175

Practice and Problem Solving: C

1. -98
2. 120
3. -144
4. 135
5. -24
6. -36
7. 0
8. -1,440
9. 1,176
10. $3(-4) = -12$; $-12 + 9 = -3$; -3 yd
11. $4(-35) = -140$; $-140 + 220 = 80$; \$80
12. $3(-50) = -150$; $-125 + (-150) = -275$; -275 ft
13. 1
14. -1
15. 1
16. -1

17. 1
18. negative; positive

Practice and Problem Solving: D

1. -6
2. 0
3. 8
4. -28
5. 12
6. -36
7. -50
8. -18
9. -70
10. 1
11. -12
12. 4
13. $5(-3) = -15$; -15 points
14. $3(-1) = -3$; -3°
15. $2(-4) = -8$; -8 yd
16. $7(-9) = -63$; -\$63
17. $5(-5) = -25$; -\$25

Reteach

1. -2
2. 18
3. -5
4. 54
5. 44
6. $4(-8) = -32$; -32 points
7. $5(-500) = -2,500$; -2,500 ft

Reading Strategies

1. gaining 10 points
2. losing 17 points
3. left
4. 4
5. left
6. 4
7. left
8. 4
9. The score decreased by 12.
10. -12 points
11. -16 points

Success for English Learners

1. -20
2. 3
3. $(-20) \times (3)$
4. $-\$60$
5. Sample answer: You know the product will be either 400 or -400 . It will be 400 because both factors are negative, so the product is positive.
6. Yes. The product of both will be negative because there is one positive factor and one negative factor. Since $4 \times 8 = 32$, each product will be -32 .

LESSON 2-2

Practice and Problem Solving: A/B

1. -12
2. 19
3. -3
4. -4
5. 11
6. -8.75
7. -5
8. -10
9. -1
10. $32 \div (-4)$
11. $\frac{-30}{6} + (-8)$
12. $12 \div (-3) + (-14)4$
13. $\$3,000 \div 40 = \75 ; $\$75 - \$40 = \$35$
14. a. $-240 \div (-15) = 16$; 16 weeks
b. $20 \times \$15 = \300 ; $\$300 - \$245 = \$55$

Practice and Problem Solving: C

1. -16
2. 2
3. $3\frac{2}{3}$
4. $+2$ produces $+2$; $+3$ produces $+6$.
5. $+2$ produces $+2$.
6. None of the integers from -3 to 3 produces a positive, even integer.

7. $+1$ produces $+2$.
8. $-16 \div 4 = -4$; -4 points for each penalty
9. a. 58°F ; $70^\circ\text{F} - (6 \text{ yd})(2^\circ\text{F}/\text{yd}) = 70^\circ\text{F} - 12^\circ\text{F} = 58^\circ\text{F}$; from 6 yd to 15 yd deep, the temperature is constant, so at 10 yd deep, the temperature is 58°F .
b. 73°F ; $50 \text{ ft} = 16\frac{2}{3} \text{ yd}$ below the surface; at 15 yd below the surface, the temperature is 58°F . But, from 15 yd to 20 yd the temperature increases 3°F per ft. $16\frac{2}{3} \text{ yd}$ is $16\frac{2}{3} - 15$ or $1\frac{2}{3} \text{ yd}$, which is 5 ft , so the temperature there is $58^\circ\text{F} + (5 \text{ ft})(3^\circ\text{F}/\text{ft})$ or $58^\circ\text{F} + 15^\circ\text{F} = 73^\circ\text{F}$.
c. $70^\circ\text{F} - (6 \text{ yd})(2^\circ\text{F}/\text{yd}) + (5)(3 \text{ ft})(3^\circ\text{F}) = 103^\circ\text{F}$ at the spring source

Practice and Problem Solving: D

1. 5
2. -9
3. -4
4. $>$
5. $<$
6. $=$
7. $-45 \div 5 = -9$
8. $\frac{55}{-11} = -5$
9. $-38 \div 19 = -2$
10. $-4 \div -2 = 2$
11. $-24 \div 4 = -6$; On average, each investor lost 6% .
12. $-760 \div 4 = -190$; On average, the temperature dropped $190^\circ/\text{h}$.
13. $-5,100 \div 3 = -1,700$; On average, the car's value decreased $\$1,700/\text{yr}$.

Reteach

1. right; negative; negative
2. left; negative; positive
3. left; positive; negative

4.

Divisor	Dividend	Quotient
+	+	+
-	+	-
+	-	-
-	-	+

Reading Strategies

- 3,600 km; 225 km/h; 16 hours
- 35 degrees; 7 hours; 5 degrees per hour
- 1,600 liters; 2-liters/bottle; 800 bottles
- Answers will vary. Sample answers: "102 divided by negative 6." "Negative 6 goes into 102 how many times?"
- Answers will vary. Sample answers: "The opposite of 17 divided into negative 221." "Negative 221 divided by negative 17."

Success for English Learners

- $\frac{-210}{70} = -3$
- $300 \overline{) -4200} = -14$
- $-50 \div 10 = -5$
- $27 \overline{) 54} = 2$
- +, 1
- , -32
- , -4
- +, 5

LESSON 2-3

Practice and Problem Solving: A/B

- 14
- 16
- 27
- 15
- 29
- 40
- >
- >
- $15(2 - 5) = -45$; \$45 less

10. $(-12) + (-11) + (-8) = -31$; falls by 31 ft

11. $5(3) + 2(-12) = -9$; 9-yd loss

12. $7(-3) + (-12) + 5 = -28$; \$28 less

Practice and Problem Solving: C

- +10
- 18
- +104
- 28
- $8(-2 + 9 + 6)$
- gained \$68
- $4(-45) + 112 = -68$; 68 ft lower
- $17(5) + 5(-2) + 8 = 83$; She got an 83.
- $3(-20) + 2(-12) + (-42) + 57 - 15 = -84$; \$84 less
- a. Positive, because there is an even number of negative factors.
b. 2,880

Practice and Problem Solving: D

- $15 + (-12)$; 3
- $15 + 18$; 33
- $-7 + 23$; 16
- $52 + (-5)$; 47
- $(-50) + (-112) + (-46) = -208$; He has \$208 less.
- $8 + (-4) + 7 + 3 + (-11) = 3$; They had a 3-yd gain.
- $4(-2) + 2(-1) + 3 = -7$; She had \$7 less.
- $3(-4) + 4(-2) = -20$; The water was 20 in. lower.

Reteach

- multiplication
- addition
- division
- addition
- multiplication
- division
- multiplication
- subtraction
- 1
- 31

11. -31
12. 33
13. -62
14. -48

Reading Strategies

1. paid; gave; $4(-3) + 7 = -12 + 7 = -5$;
\$5 less
2. below; $-48 \div 4 = -12$; 12 feet below the surface
3. lost; gained; $3(-5) + 32 = -15 + 32 = 17$;
gained 17 yards

Success for English Learners

1. 39
2. -5
3. 6
4. a. Sample answer: Tom bought 3 DVDs for \$20 each. He had a coupon for \$5 off one DVD. After his purchase, what is the change in the amount of money Tom has?
b. $-3(20) + 5 = -60 + 5 = -55$; Tom has \$55 less now.

MODULE 2 Challenge

1. Sample answer:
 $81 \div (-9) + (-4) - 17 + (4)(3) + 1$
 $-9 + (-4) - 17 + 12 + 1$
 $-13 - 17 + 12 + 1$
 $-30 + 12 + 1$
 $-18 + 1$
 -17
2. Sample answer: Play with 2–4 players. Shuffle the integer cards and deal them out. Place the operations card face-up on the table. One player starts making an expression by placing one card on the table. The next player can choose an operation card and an integer card from his/her hand and extend the expression. Each player does the same until the cards are gone or one player wins. To win, a player makes the expression equal to 0.

3. Sample answer:

First find multiplication and division signs and do these operations first.

1. Multiply $(-4)(7) = -28$. The product is negative because one of the factors is negative.

$$(-8) + (-3) + (-28) \div 14 + 9(-2)$$

2. Divide $(-28) \div 14 = -2$. The quotient is negative because the dividend is negative and the divisor is positive.

$$(-8) + (-3) + (-2) + 9(-2)$$

3. Multiply $(9)(-2) = -18$. Same reason as step 1.

$$(-8) + (-3) + (-2) + (-18)$$

Now go back and add and subtract from left to right.

4. $(-8) + (-3) = (-11)$ because you are adding two negative numbers.

$$(-11) + (-2) + (-18)$$

5. $(-11) + (-2) = (-13)$, for the same reason. $(-13) + (-18)$

$$6. (-13) + (-18) = (-31)$$

MODULE 3 Rational Numbers

LESSON 3-1

Practice and Problem Solving: A/B

1. 0.95
2. -0.125
3. 3.4
4. $-0.777\dots$ or $0.\overline{7}$
5. $0.7333\dots$ or $0.7\overline{3}$
6. $2.666\dots$ or $2.\overline{6}$
7. $\frac{29}{9}$; $3.222\dots$; repeating or $3.\overline{2}$
8. $\frac{301}{20}$; 15.05; terminating
9. $-\frac{53}{10}$; -5.3; terminating

10. a. Answers may vary. Sample answer:

$$2\frac{3}{4}, 2.75; 3\frac{2}{4}, 3.5$$

b. Answers may vary. Sample answer:

$$4\frac{2}{3}, 4.666\dots \text{ or } 4.\bar{6}$$

11. They all convert to terminating decimals.

Practice and Problem Solving: C

1. $\frac{25}{18}$; 1.3888... or $1.3\bar{8}$; repeating

2. $\frac{200}{15}$; 13.333... or $13.\bar{3}$; repeating

3. Possible answer: $\frac{5}{20}, \frac{18}{20}, \frac{3}{20}$; the decimals are 0.25, 0.9, 0.15. They terminate because a rational number with 20 in the denominator is equivalent to a rational number with 100 in the denominator, which always terminates.

4. Possible answer: $\frac{30}{15} = 2.0$; $\frac{5}{15} = 0.333\dots$

or $0.\bar{3}$; To find a repeating decimal, select a multiple of 5 that is less than 15. To find a terminating decimal, select a numerator that is a multiple of 15.

5. Possible answer: Yes; $\frac{1.5}{7.5} = \frac{15}{75}$, which is written as a ratio of two integers;

$$\frac{15}{75} = 0.2$$

Practice and Problem Solving: D

1. 0.65; terminating

2. 4.666... or $4.\bar{6}$; repeating

3. 0.555... or $0.\bar{5}$; repeating

4. 3.833... or $3.8\bar{3}$; repeating

5. 8.75; terminating

6. 10.625; terminating

7. 1.3125

8. 7.3125

9. 26.3125

10. 1.266... or $1.2\bar{6}$

11. 17.266... or $17.2\bar{6}$

12. 23.266... or $23.2\bar{6}$

Reteach

1. $\frac{3}{4} = 0.75$ so $7\frac{3}{4} = 7.75$

2. $\frac{5}{6} = 0.833\dots$ or $0.8\bar{3}$ so $11\frac{5}{6} = 11.833\dots$
or $11.8\bar{3}$

3. $\frac{3}{10} = 0.3$ so $12\frac{3}{10} = 12.3$

4. $\frac{5}{18} = 0.277\dots$ or $0.2\bar{7}$ so $8\frac{5}{18} = 8.277\dots$
or $8.2\bar{7}$

5. Sample answer:

Method 1: Start with the fraction part.

$$\frac{2}{9} = 0.222\dots \text{ or } 0.\bar{2} \text{ so } 9\frac{2}{9} = 9.222\dots \text{ or } 9.\bar{2}$$

Method 2: $9\frac{2}{9} = \frac{83}{9}$. Using long division,

$$\frac{83}{9} = 9.222\dots \text{ or } 9.\bar{2}; \text{ the results agree.}$$

6. Sample answer:

Method 1: Start with the fraction part.

$$\frac{5}{8} = 0.625 \text{ so } 21\frac{5}{8} = 21.625.$$

Method 2: $21\frac{5}{8} = \frac{173}{8}$. Using long

division, $\frac{173}{8} = 21.625$; the results agree.

Reading Strategies

1. Both -3 and 5 are integers.

2. 2 is an integer but 1.17 is not an integer (but that does not mean that $\frac{2}{1.17}$ is not a rational number).

3. 1 is an integer but $\frac{1}{3}$ is not an integer

(but that does not mean that $\frac{1}{\frac{1}{3}}$ is not a rational number).

4. $\sqrt{2}$ is not an integer and $\sqrt{4}$ is not an integer (but $\sqrt{4}$ can be written as the integer 2).

5. $\frac{\sqrt{3}}{\sqrt{6}}$ cannot be written as a ratio of two integers.
6. $\frac{\sqrt{2}}{\sqrt{2}} = 1$, so it can be written as the ratio of two integers such as $\frac{1}{1}$ or $\frac{-3}{-3}$.
7. $\frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$, so it can be written as the ratio of two integers.
8. $\frac{\sqrt{1}}{2} = \frac{1}{2}$, so it can be written as the ratio of two integers.

Success for English Learners

1. D
2. B
3. Answers may vary. Sample answer: 4.616161...; $\overline{4.61}$

LESSON 3-2

Practice and Problem Solving: A/B

1. 1
2. -7
3. 9
4. $-2\frac{1}{2}$
5. $\frac{1}{9}$
6. -8.4
7. $-5\frac{1}{2}$
8. -3.1
9. $-\frac{11}{20}$
10. -3.3
11. 2.46
12. -1.85
13. -6.85
14. $-3\frac{1}{8}$
15. \$3.75

16. gain of 6
17. \$6.85
18. 3.8 mi from his house

Practice and Problem Solving: C

1. 4
2. $2\frac{16}{45}$
3. $8\frac{19}{45}$
4. -7.6
5. $-2\frac{1}{8}$
6. -1.13
7. $-\frac{59}{180}$
8. 0
9. 9.929
10. -2.278
11. -1.75
12. $1\frac{3}{23}$
13. \$8.07
14. \$5.57
15. 5 in.

Practice and Problem Solving: D

1. 5
2. -1
3. -8
4. 3
5. 8
6. -6
7. 0.5
8. -2.0
9. 2
10. $-\frac{1}{2}$
11. 2
12. 0
13. 4

14. $2\frac{1}{3}$
15. $\frac{3}{4}$
16. -3.4
17. -3.2
18. -0.5
19. $-1\frac{1}{2}$
20. -3
21. -0.9

Reteach

1. 2
2. -5
3. -7
4. 0.6
5. 4.7
6. -6
7. $\frac{3}{5}$
8. $-1\frac{2}{3}$
9. $-\frac{1}{2}$

Reading Strategies

1. 0
2. to the right; 6
3. to the left; 4
4. 0
5. to the left; 5.5
6. to the left; 3

Success for English Learners

1. Answers will vary. Sample answer: so the digits of the same place value get added together
2. the total number of slices of pizza

LESSON 3-3

Practice and Problem Solving: A/B

1. -9
2. 9
3. 9
4. $-5\frac{1}{2}$
5. $-\frac{2}{7}$
6. 1.2
7. $\frac{3}{4}$
8. -3.7
9. $-5\frac{1}{2}$
10. 8.3
11. -9.08
12. 3.75
13. -6.2
14. $-1\frac{3}{5}$
15. -4.1°C
16. $1\frac{3}{5}$ m

Practice and Problem Solving: C

1. $-6\frac{2}{3}$
2. $1\frac{1}{21}$
3. -10
4. -7.2
5. $-2\frac{1}{8}$
6. -12.179
7. $-1\frac{5}{9}$

- 8. 0.36
- 9. -13.19
- 10. -4.35
- 11. -1.05
- 12. -7
- 13. 3.55
- 14. Alex by 7.1 points
- 15. 7°C

Practice and Problem Solving: D

- 1. 2
- 2. 6
- 3. -3
- 4. -7
- 5. -3
- 6. 8
- 7. 1.5
- 8. -3
- 9. -1.5
- 10. $1\frac{1}{2}$
- 11. -1
- 12. $-1\frac{1}{2}$
- 13. 7
- 14. $-\frac{4}{3}$ or $-1\frac{1}{3}$
- 15. $-\frac{1}{2}$
- 16. 1.4
- 17. -2.2
- 18. -7.8
- 19. -2
- 20. -6.5
- 21. -1

Reteach

- 1. a. 5
b. -1
c. 20
- 2. a. negative
b. 2
c. -2

- 3. 40
- 4. -3
- 5. -26
- 6. 4.2
- 7. 2
- 8. -3.25
- 9. 1
- 10. -2
- 11. $-\frac{5}{4}$

Reading Strategies

- 1. Sample answer: One number is placed in each square.
- 2. as a placeholder to show that there is no number in that place
- 3.

4	0	•	3	
-	6	•	5	4

- 4. yes; in the hundredths place of the first number
- 5. 33.76

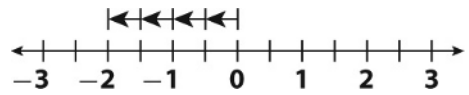
Success for English Learners

- 1. -9
- 2. You are not adding or subtracting -4, you are subtracting 3 from -4.
- 3. No, in $3 - 5$ you are subtracting 5 (or adding -5) to 3. In $5 - 3$ you are subtracting 3 from 5.
- 4. Find a common denominator
- 5. $\frac{2}{15}$

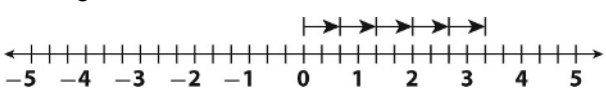
LESSON 3-4

Practice and Problem Solving: A/B

- 1. -2



- 2. $3\frac{1}{3}$



3. -6.2
4. -21.6
5. -19.8
6. 16.8
7. 36
8. -2.1
9. -8.2
10. 31.5
11. -20
12. $-\frac{4}{9}$
13. 9
14. $\frac{1}{2}$
15. $12\left(\frac{3}{4}\right) = 9$; 9 yards
16. $\left(\frac{1}{4}\right)\left(\frac{2}{3}\right)\left(\frac{3}{5}\right) = \frac{1}{10}$; $\frac{1}{10} \text{ m}^3$
17. $(-3 \text{ }^\circ\text{F/half hour}) \times (2 \text{ half hours/hour}) \times 4 \text{ hours} = -24 \text{ }^\circ\text{F}$; $75 \text{ }^\circ\text{F} - 24 \text{ }^\circ\text{F} = 51 \text{ }^\circ\text{F}$

Practice and Problem Solving: C

1. <; The product of 3 positive numbers, each of which is less than 1, is less than 1.
2. <; The product of 3 negative numbers is a negative number.
3. >; The product of 3 positive numbers is greater than the product of the opposite of each of the positive numbers.
4. <; the product of a positive and a negative number is less than 0.
5. False; A negative number raised to an even power is a positive number.
6. True; A number that is greater than 1 raised to a positive power is greater than 1.
7. False; A positive number that is less than one raised to a power is less than 1.

$$8. V_1 = \frac{4}{3}\pi\left(\frac{1}{2}\right)^3 = \frac{4\pi}{24} = \frac{\pi}{6} \text{ ft}^3;$$

$$V_2 = \frac{4}{3}\pi\left(\frac{3}{4}\right)^3 = \frac{108\pi}{192} = \frac{9\pi}{16} \text{ ft}^3; V_2 > V_1,$$

since $\frac{9\pi}{16} = 0.5625\pi$ and $\frac{\pi}{6} = 0.16\bar{6}\pi$.

9. $V = \frac{4}{3}\pi r^3$. If r becomes $\frac{2r}{3}$, then

$$V_2 = \frac{4}{3}\pi\left(\frac{2r}{3}\right)^3 = \frac{8}{27}\left(\frac{4}{3}\pi r^3\right).$$

Therefore, if the radius is reduced to one third of its original value, the volume is $\frac{8}{27}$ or 0.296 of the original volume.

Practice and Problem Solving: D

1. $\left(-\frac{1}{2}\right)$; $\left(-\frac{1}{2}\right)$; $\left(-\frac{1}{2}\right)$; $\left(-\frac{1}{2}\right)$; $\left(-\frac{1}{2}\right)$;
 $\left(-\frac{1}{2}\right)$; $-\frac{6}{2}$ or -3
2. $\left(-\frac{2}{3}\right)$; $\left(-\frac{2}{3}\right)$; $\left(-\frac{2}{3}\right)$; $\frac{6}{3}$ or 2
3. Answers may vary. Sample answer:
 $4\left(-\frac{5}{8}\right)$; $\frac{20}{8}$ or $\frac{5}{2}$ or $-2\frac{1}{2}$
4. Answers may vary. Sample answer:
 $2(-2.5)$; -5
5. Answers may vary. Sample answer:
 $3\left(-\frac{2}{9}\right)$; $-\frac{2}{3}$
6. $-\frac{1}{4} \times \left(\frac{-6}{25}\right) = \frac{6}{100} = \frac{3}{50}$ or 0.06
7. $4 \times 2.5 \times 0.8 = 10 \times 0.8 = 8$
8. a. $(-3.5) + (-3.5) + (-3.5) + (-3.5) + (-3.5) = -17.5 \text{ m}$; -17.5 m
b. $5 \times (-3.5) = -17.5$; -17.5 m

Reteach

1. 6; right; $\frac{6}{4}$; $1\frac{1}{2}$
2. 8 times; 26.4; 26.4
3. 5 times; 23; 23

Reading Strategies

- 4
- 5.25
- $-2\frac{2}{5}$

Success for English Learners

- 8.8
- 3
- 9.9
- 9.9
- 12
- $\frac{12}{7}$ or $1\frac{5}{7}$
- 4

LESSON 3-5

Practice and Problem Solving: A/B

- $-\frac{1}{6}$
- 8
- $\frac{1}{12}$
- 0.35
- 7.5
- 0.25
- $\frac{2}{3}$
- $-\frac{4}{3}$
- $-\frac{9}{20}$
- 6
- 1.75
- 2
- 1
- $\frac{3}{14}$
- $\frac{1}{98}$
- $8 \div \frac{1}{4}$; 32 packets

- $\frac{3}{4} \div 12$; $\frac{1}{16}$ h
- $\frac{35}{1.25}$; 28 pieces
- $4\frac{1}{8} \div 2\frac{1}{6} = \frac{99}{52}$ or $1\frac{47}{52}$ tons per acre

Practice and Problem Solving: C

- $\frac{1}{-5}$ and $-\frac{1}{5}$
- $\frac{-7}{30}$ and $\frac{7}{-30}$
- $\frac{-1}{2}$ and $-\frac{1}{2}$
- =; $\frac{10}{3}$; $\frac{10}{3}$
- <; $(4.5 \div 0.5) \div 3 = 3$; $4.5 \div (0.5 \div 3) = 27$
- >; $\left(6 \div -\frac{1}{5}\right) \times -\frac{4}{3} = 40$;
 $6 \div \left(-\frac{1}{5} \times -\frac{4}{3}\right) = 22.5$
- =; -123.75; -123.75
- $\frac{0.4}{-0.625} = -0.64$
- $\frac{-5.4}{-0.3125} = 17.28$
- $\frac{0.25}{0.6} = 0.4\overline{16}$
- $\frac{\left(\frac{1}{4}\right)}{\frac{1}{4}} = 7$; $\frac{1.75}{0.25} = 7$
- $\frac{\left(\frac{1}{2}\right)}{\frac{1}{2}} = 3$; $\frac{1.5}{0.5} = 3$
- $\frac{\left(\frac{3}{8}\right)}{\frac{5}{8}} = 2\frac{1}{5}$; $\frac{1.375}{0.625} = 2.2$
- The quotient will be less than 7, 3, and 2.2 but greater than 1. It decreases as the denominators of the fractions increase.

Practice and Problem Solving: D

- $\frac{4}{3}; -8$
- $\frac{1}{8}; \frac{1}{10}$
- $\frac{-4}{7}; \frac{1}{2}$
- $\frac{8}{7}; \frac{-40}{21} = -1\frac{19}{20}$
- $\frac{9}{4}; \frac{-9}{2}$
- $\frac{1}{4}; -1\frac{3}{16}$
- $\frac{1}{40}$
- $\frac{-21}{8} = -2\frac{5}{8}$
- $\frac{7}{2} = 3\frac{1}{2}$
- 0.40; 0.16
- 0.30; -15.83
- 8.0; 3.2
- a. $6\frac{3}{4} \div \frac{1}{8}$
b. 54 markers
c. The town spaced the markers every eighth of a mile. They used 54 markers. Since $6\frac{3}{4}$ is evenly divisible by $\frac{1}{8}$, they used a whole number of markers.

Reteach

- +
-
-
- +
- $-\frac{1}{7} \div -\frac{5}{9} = -\frac{1}{7} \times -\frac{9}{5}; -\frac{1}{7} \times -\frac{9}{5} = \frac{-9}{-35}; \frac{-9}{-35} = \frac{9}{35}$

A negative divided by a negative is positive.

- $\frac{7}{8} \div \frac{8}{9} = \frac{7}{8} \times \frac{9}{8}; \frac{7}{8} \times \frac{9}{8} = \frac{63}{64};$
 $\frac{63}{64}$ is positive since a positive divided by a positive is positive.

Reading Strategies

- +
-
- +
-
-
- +
-
- +
- +
-
- +
-
-
-
- +

Success for English Learners

- $2\frac{7}{88}$
- 2

LESSON 3-6

Practice and Problem Solving: A/B

- Answers may vary. Sample answer: One estimate would be 4 times 6 or 24 feet long. The actual answer is greater than 24 feet.
- Answers may vary. Sample answer: 3 liters divided by a third of a liter makes about 9 servings. The actual answer is more than 9 servings.
- Answers may vary. Sample answer: The perimeter is greater than 15 inches.
- Answers may vary. Sample answer: 3-gram eggs would be 36 grams, but 4 gram eggs would be 48 grams, so a dozen 3.5-gram eggs should be about 42 grams.

5. Answers may vary. Sample answer:
8 divided by one half is 16, so the number of peas is greater than 16.
6. These numbers can be used as they are since there would be 8 drops in a milliliter, or 240 drops in 30 milliliters.
7. The second strip is 0.25 longer than 3.5, or $3.5 + 0.875$, or 4.375 yards. The length of the third strip can be written as 6.25, so the total length is $3.5 + 4.375 + 6.25$, or 14.125 yards. 0.125 yards is one eighth of a yard, so the answer might be written as $14\frac{1}{8}$ yd.

Practice and Problem Solving: C

1. $29\frac{37}{50}$ m/s \times 3,600 s/h = 107,064 mi
2. $29\frac{37}{50} - 8\frac{3}{25} = 29\frac{37}{50} - 8\frac{6}{50} = 21\frac{31}{50}$ mi/s
3. $32,508$ mi \div $6\frac{2}{100}$ mi/s = 5,400 s
4. $21\frac{19}{25}$ mi/s \times 60 s/min = $1,305\frac{3}{5}$ mi/min

Practice and Problem Solving: D

1. Bottles, paper, and cardboard boxes were $\frac{11}{20}$ of the total amount of recycled material collected by the middle school.
2. $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{3} = \frac{2}{6}$; $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$; $\frac{5}{6}$ of the family budget

$$3. \frac{1}{6} = \frac{4}{24}, \frac{3}{8} = \frac{9}{24}; \frac{4}{24} + \frac{9}{24} = \frac{13}{24};$$

$$1 = \frac{24}{24}; \frac{24}{24} - \frac{13}{24} = \frac{11}{24} \text{ of the budget}$$

Reteach

1. $11\frac{2}{5}$ oz
2. 8 h
3. $15\frac{2}{5}$ t
4. $1\frac{1}{16}$ lb

Reading Strategies

1. $2\frac{1}{2}$ feet
2. $\frac{1}{2}$ foot
3. 5 servings
4. 5
5. 5 ft
6. 5
7. Answers may vary, but students should observe that the answers are the same, and divisor is the reciprocal of the factor 2.

Success for English Learners

1. the number of pieces of pizza
2. Find the common denominator.
3. Add the numerators, and write the sum over the common denominator.

MODULE 3 Challenge

1. Calculate the daily temperature change as shown.

Daily Temperature Change ($^{\circ}$ C)

City	Monday to Tuesday	Tuesday to Wednesday	Wednesday to Thursday	Thursday to Friday
City A	$2\frac{1}{4} - \left(-\frac{1}{8}\right) = 2\frac{3}{8}$	$-3\frac{1}{2} - 2\frac{1}{4} = -5\frac{3}{4}$	$5\frac{4}{5} - \left(-3\frac{1}{2}\right) = 9\frac{3}{10}$	$-12\frac{1}{2} - 5\frac{4}{5} = -18\frac{3}{10}$
City B	$-1\frac{3}{5} - 4\frac{1}{5} = -5\frac{4}{5}$	$-8\frac{1}{10} - 1\frac{3}{5} = -6\frac{1}{2}$	$11\frac{1}{5} - \left(-8\frac{1}{10}\right) = 19\frac{3}{10}$	$3\frac{3}{10} - 11\frac{1}{5} = -7\frac{9}{10}$
City C	$2\frac{5}{6} - 11\frac{1}{3} = -8\frac{1}{2}$	$-3\frac{2}{3} - 2\frac{5}{6} = -6\frac{1}{2}$	$-9\frac{1}{6} - \left(-3\frac{2}{3}\right) = -5\frac{1}{2}$	$2\frac{1}{3} - \left(-9\frac{1}{6}\right) = 11\frac{1}{2}$

Find the sum of the daily temperature changes for each city.

$$\text{City A: } 2\frac{3}{8} + -5\frac{3}{4} + 9\frac{3}{10} - 18\frac{3}{10} = -12\frac{3}{8}$$

$$\text{City B: } -5\frac{4}{5} + \left(-6\frac{1}{2}\right) + 19\frac{3}{10} + \left(-7\frac{9}{10}\right) = -\frac{9}{10}$$

$$\text{City C: } -8\frac{1}{2} + -6\frac{1}{2} + -5\frac{1}{2} + 11\frac{1}{2} = -9$$

The greatest temperature volatility is in City B.

2. There are 24 ways to place the four operations in the three different boxes:

$$2 \boxed{+} -\frac{1}{8} \boxed{-} -10 \boxed{\times} 16 = 161.875; 2 \boxed{+} -\frac{1}{8} \boxed{-} -10 \boxed{\div} 16 = 2.5;$$

$$2 \boxed{+} -\frac{1}{8} \boxed{\times} -10 \boxed{-} 16 = -12.75; 2 \boxed{+} -\frac{1}{8} \boxed{\times} -10 \boxed{\div} 16 = 2.078125;$$

$$2 \boxed{+} -\frac{1}{8} \boxed{\div} -10 \boxed{-} 16 = -13.9875; 2 \boxed{+} -\frac{1}{8} \boxed{\div} -10 \boxed{\times} 16 = 2.2;$$

$$2 \boxed{-} -\frac{1}{8} \boxed{+} -10 \boxed{\times} 16 = -157.875; 2 \boxed{-} -\frac{1}{8} \boxed{+} -10 \boxed{\div} 16 = 1.5;$$

$$2 \boxed{-} -\frac{1}{8} \boxed{\times} -10 \boxed{+} 16 = 16.75; 2 \boxed{-} -\frac{1}{8} \boxed{\times} -10 \boxed{\div} 16 = 1.921875;$$

$$2 \boxed{-} -\frac{1}{8} \boxed{\div} -10 \boxed{+} 16 = 17.9875; 2 \boxed{-} -\frac{1}{8} \boxed{\div} -10 \boxed{\times} 16 = 1.8;$$

$$2 \boxed{\times} -\frac{1}{8} \boxed{+} -10 \boxed{-} 16 = -26.25; 2 \boxed{\times} -\frac{1}{8} \boxed{+} -10 \boxed{\div} 16 = -0.875;$$

$$2 \boxed{\times} -\frac{1}{8} \boxed{-} -10 \boxed{+} 16 = 25.75; 2 \boxed{\times} -\frac{1}{8} \boxed{-} -10 \boxed{\div} 16 = 0.375;$$

$$2 \boxed{\times} -\frac{1}{8} \boxed{\div} -10 \boxed{+} 16 = 16.025; 2 \boxed{\times} -\frac{1}{8} \boxed{\div} -10 \boxed{-} 16 = -15.975;$$

$$2 \boxed{\div} -\frac{1}{8} \boxed{+} -10 \boxed{-} 16 = -26.25; 2 \boxed{\div} -\frac{1}{8} \boxed{+} -10 \boxed{\times} 16 = -176;$$

$$2 \boxed{\div} -\frac{1}{8} \boxed{-} -10 \boxed{+} 16 = 10; 2 \boxed{\div} -\frac{1}{8} \boxed{-} -10 \boxed{\times} 16 = 144;$$

$$2 \boxed{\div} -\frac{1}{8} \boxed{\times} -10 \boxed{+} 16 = 176; 2 \boxed{\div} -\frac{1}{8} \boxed{\times} -10 \boxed{-} 16 = 144.$$

$2 \boxed{\div} -\frac{1}{8} \boxed{\times} -10 \boxed{+} 16 = 176$ is the greatest possible value of the expression. To obtain the answer without writing out all 24 possibilities, students can notice that dividing by a fraction will increase the value of the expression and that multiplying two negatives will create a positive.